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Indicators of quantum chaos based on eigenvector statistics

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Abstract. Expansion coefficients of the coherent states in the basis of Hamiltonian eigenstates contain information about the local character of motion of a quantum system. We analyse three quantities: the minimal number of relevant eigenvectors, the sum of moduli of coefficients and the Shannon entropy of a coherent state, and show that all of them might be used as indicators of quantum chaos. They also allow us to distinguish between three known universality classes. Results obtained are confirmed by a numerical study of kicked tops.

1. Introduction

During the last decade an enormous amount of work was done in order to elucidate the quantum dynamics of classically chaotic systems (see [1] and references therein). Several ways in which chaotic motion manifests itself in quantum mechanics were found. In particular the level and eigenvector statistics of quantum analogues of classically chaotic systems conforms to the predictions of random-matrix theory [2-4].

Not so much is known about the 'local' features of quantum chaotic systems. The transition from regular to chaotic motion in classical systems does not occur homogeneously: the volume of stochastic layers increases gradually in phase space [5]. It is therefore interesting to study such quantities of the quantum system which allow for an analysis of the quantum-classical correspondence. Local properties of quantum systems were usually investigated with help of coherent states (or any other localized wavepackets). One approach is based on the Husimi distribution or the Q function, i.e. the expansion of an eigenstate of the dynamics in the overcomplete basis of coherent states [6-9].

A complementary picture can be obtained by studying the expansion of a coherent state (CS) in the orthonormal basis of the eigenstates of the Hamiltonian (or the Floquet operator in case of time-dependent periodic systems). Statistical properties of components of a particular coherent state in this basis are connected with the quantum dynamics. In particular, the distribution of components is in general more homogeneous for CS localized in chaotic layers. In order to facilitate the quantitative analysis a quantity called number of relevant eigenstates M was introduced and discussed [10]. M can be defined as a minimal number of eigenstates exhausting the normalization of CS up to a given reference value r. Numerical calculations have shown that the number M was large for a CS localized in the classical phase space in a region of chaotic motion. It was therefore conjectured that a correlation between the number M and the Lapunov exponent Λ exists.

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In this work we support this conjecture with additional arguments and study two other quantities which provide equivalent characterization of the local properties of a quantum system: the sum of moduli of expansion coefficients S and the Shannon entropy of coherent state H_s. The paper is organized as follows. In section 2 we discuss whether the statistics of components of CS in the eigenbasis of the Hamiltonian is given by the χ_ν^2 distribution, and the consequences of this assumption. In particular analytical formulae for S and H_s are derived. In section 3 generalized quantities $S(e)$ and $H(u)$ are defined by allowing for suitable free parameters e and u vaguely analogous to the reference level r in $M(r)$. Each family $M(r)$, $S(e)$ and $H(u)$ provides reasonable chaos indicators since the functional dependence of each quantity on its parameter is smooth and without fluctuations. In section 4 periodically kicked tops are defined and the numerical results are presented. We show how chaos indicators work in a quantum system the classical analogue of which displays regular and chaotic behaviour in its phase space. In the case of full developed classical chaos systems pertaining to all three known universality classes are analysed. Concluding remarks are contained in section 5.

2. Expansion of coherent state in eigenbasis of the system

Consider an arbitrary integrable system H_0 with eigenstates $|l\rangle$, $l=1,\ldots,N$ and the perturbed system $H=H_0+\lambda V$ with eigenstates $|\phi_i\rangle$, $i=1,\ldots,N$. For a perturbation parameter λ large enough the system may be described by a Gaussian ensemble of random matrices (or circular in the case of periodic systems). Depending on the system symmetry the orthogonal (OE), unitary (UE), or symplectic ensemble (SE) should be applied. The distribution of components $y=|\langle l|\phi_i\rangle|^2$ of any eigenvector $|\phi_i\rangle$ of the OE tends in the limit of $N\to\infty$ to the Porter-Thomas distribution [11]:

$$P(y)=(2\pi\langle y\rangle y)^{-1/2}\exp(-y/2\langle y\rangle) \tag{2.1}$$

where the mean value $\langle y\rangle$ is equal to N^{-1}. This formula is a particular case of the χ_ν^2 distribution

$$P_\nu(y)=\left(\frac{\nu}{2\langle y\rangle}\right)^{\nu/2}\frac{1}{\Gamma(\nu/2)}y^{(\nu/2)-1}\exp\left(\frac{-\nu y}{2\langle y\rangle}\right) \tag{2.2}$$

with $\nu=1$ degree of freedom. Let me briefly recall the basic property of this distribution. Consider ν independent random variables x_i, $i=1,\ldots,\nu$. If each variable has the Gaussian probability distribution with the mean equal to zero and the variance equal to $\sigma/\sqrt{\nu}$ then the sum of squares $y=\Sigma_{i=1}^{\nu}x_i^2$ will obey the χ_ν^2 distribution with mean value $\langle y\rangle=\sigma^2$.

The χ_ν^2 distribution describes the eigenvector statistics also for the unitary ensemble with ν equal to 2 and for the symplectic ensemble with ν equal to 4 [12-14]. It can be understood as a simple consequence of the fact that the OE is built of real matrices, whereas in unitary ensemble the real and imaginary parts of each complex element of a matrix are two independent variables. Since two complex numbers form a basic element of the symplectic matrix, the χ^2 distribution with four degrees of freedom is adequate in this case. It was also conjectured that the χ^2 distribution with $\nu\to 0$ approximates the eigenvector statistics if the perturbation parameter λ decreases and system becomes regular [13, 14]. No rigorous proof was given, but the numerical results do not contradict this statement.

The expansion of some fixed vector $|l\rangle$ in the basis of eigenstates $|\phi_i\rangle$ will obviously have the same statistical properties. In a pure chaotic case, described by random matrices, the only characteristic of a vector is its own norm. Thus for a large enough size of matrix N the components $y=|\langle\alpha|\phi_i\rangle|^2$ of an arbitrary coherent state $|\alpha\rangle$ in the eigenbasis $|\phi_i\rangle$, $i=1,\ldots,N$ will also conform to the χ^2_ν distribution with $\nu=1$, 2 or 4 degrees of freedom, depending on the system symmetry.

It was shown [10] that a CS localized in the region of classically regular motion has a few relevant components in the eigenbasis $|\phi_i\rangle$. This feature is characteristic for the χ^2_ν distribution with $0<\nu\ll 1$. It is therefore interesting to analyse consequences of the assumption that the χ^2_ν distribution holds for the CS also in the case of transition from chaotic to regular motion.

Let $|\alpha_{p,q}\rangle$ be a CS localized in point $\{p, q\}$ of the classical phase space and let its expansion be

$$|\alpha_{p,q}\rangle=\sum_{i=1}^{N} c_i|\phi_i\rangle. \tag{2.3}$$

The minimal number of relevant eigenstates $M(r)$ is defined [10] as

$$M(r)=\min\left\{M:\ \sum_{i=1}^{M}|c_i|^2>r;\ |c_i|\geq|c_{i+1}|\right\}. \tag{2.4}$$

In [10] the parameter r was set arbitrarily to 0.99. The quantity M seems to be correlated with the classical Lapunov exponent: M is large for a CS localized in a chaotic layer. We suggest studying two other quantities as well: the sum of moduli S

$$S:=\sum_{i=1}^{N}|c_i| \tag{2.5}$$

and the Shannon entropy H_s of the coherent state

$$H_s:=-\sum_{i=1}^{N}|c_i|^2\ln|c_i|^2. \tag{2.6}$$

The quantities defined above can be easily evaluated for components $|c_i|$ described by the χ^2_ν distribution. The sum S is (integration should be performed from $y=0$ to $y=1$, but assuming $N\gg 1$ the domain may be extended to infinity)

$$S=N\langle|c_i|\rangle\approx N\int_0^\infty \sqrt{y}\,P_\nu(y)\,\mathrm{d}y=\sqrt{N}\,G(\nu) \tag{2.7}$$

where the function G reads in terms of Euler Γ function

$$G(\nu)=\sqrt{\frac{2}{\nu}}\frac{\Gamma[(\nu+1)/2]}{\Gamma(\nu/2)}. \tag{2.8}$$

For different ν values, corresponding to different universality classes or various chaos conditions in the classical phase space the function $G(\nu)$ takes quite different values: $G(4)=3/4\sqrt{\pi/2}\approx 0.94$, $G(2)=\sqrt{\pi}/2\approx 0.89$, $G(1)=\sqrt{2/\pi}\approx 0.80$, $\lim G(\nu)_{\nu\to 0}=0$. Hence the sum S can be used to distinguish between different regions of the phase space of the very same system.

An analogous result may be obtained for the Shannon entropy of a single coherent state

$$H_s\approx -N\int_0^\infty y\ln(y)P_\nu(y)\,\mathrm{d}y=\ln\left(N\frac{\nu}{2}\right)-\Psi\left(\frac{\nu}{2}+1\right) \tag{2.9}$$

where Ψ is the digamma function. For a constant matrix dimension N the Shannon entropy changes significantly with the 'chaos' parameter ν. The particular cases of (2.9) with ν equal to 1 and 4 were recently used to analyse the localization of quasi-energy eigenstates [15, 16].

3. One-parameter families of indicators

The definition of the number of relevant eigenstates involves a parameter r without the physical meaning. The quantity M_r can not be expressed explicitly as the sum S and entropy H_s, but might be found as a solution of a transcendental equation. For a given value of the parameter r let us define x such that

$$\int_x^\infty y P_\nu(y)\,\mathrm{d}y = r\langle y\rangle. \tag{3.1}$$

The fraction M_r/N will be then given by

$$\frac{M_r}{N} \approx \int_x^\infty P_\nu(y)\,\mathrm{d}y. \tag{3.2}$$

The above integrals can be expressed by the incomplete Γ function. Equation (3.1) reads:

$$\Gamma\left(\frac{\nu}{2}+1, x\right) = r\frac{\nu}{2}\Gamma\left(\frac{\nu}{2}\right) \tag{3.3}$$

and allows us to calculate the value of x as a function of r. The number of relevant eigenvectors M_r is thus estimated by

$$M_r = \mathrm{int}\left[N\frac{\Gamma(\nu/2, x)}{\Gamma(\nu/2)}\right] + 1 \tag{3.4}$$

where int[s] denotes the integer part of a real number s.

Equations (3.3) and (3.4) allow us to compute numerically the number M as a function of the parameter r. Such dependencies for $\nu = 4$ (symplectic case), $\nu = 2$ (unitary case), $\nu = 1$ (orthogonal case) and $\nu = 0.5$ (transition chaos–regular motion) are depicted in figure 1(a). The vertical broken line is drawn at $r = 0.99$, i.e. the value used in [10]. This value, chosen arbitrarily, is as good as any other, except that r should not be larger than, say, 0.995 since the ν-sensitivity of M_r would be lost. Relative weak and smooth dependence of the number M on the parameter r verifies that this quantity is well defined. For three universality classes described by the χ^2_ν distribution with $\nu = 1$, 2 and 4 the ratio $M_{0.99}/N$ is equal to 0.74, 0.86 and 0.93, respectively.

The number of relevant eigenstates M depends on the free parameter r. One might thus consider the other indicators S and H_s as more adequate for our purposes, since they do not have this 'disadvantage'. The following quantities can be defined, however: the generalized sum S_e; $0 < e$; $e \neq 2$

$$S_e = \sum_{i=1}^N |c_i|^e \tag{3.5}$$

and the generalized Shannon entropy H_u; $0 < u$

$$H_u = -\sum_{i=1}^N |c_i|^{2u} \ln|c_i|^{2u}. \tag{3.6}$$

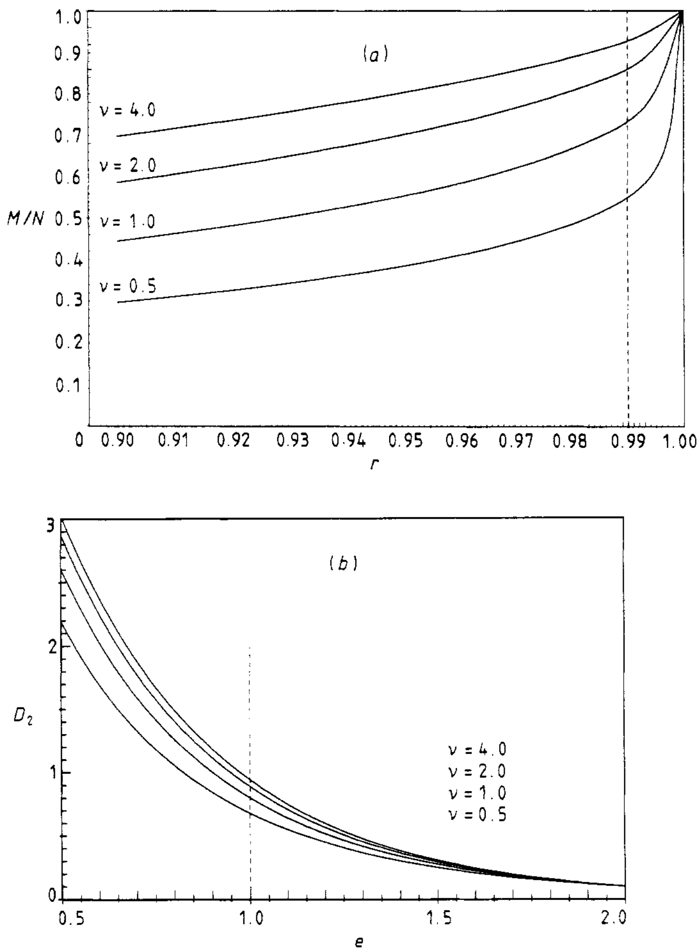


Figure 1. Dependencies of analysed quantities on the unphysical parameters for $\nu = 4$, 2, 1 and 0.5. (*a*) Number of relevant eigenstates $M(r)$. (*b*) Scaled generalized sum $D_2 = S(e)\sqrt{N}$. Broken vertical lines show the usual values of parameters.

The particular cases of (3.5) and (3.6) with $e = u = 1$ have been considered in section 3. We can take, however, any other value of the parameters e and u, e.g. $e = u = 1.1$, and the new quantities $S_{1.1}$ and $H_{1.1}$ are as good for our aim as $S_{1.0}$ and $H_{1.0}$ (nevertheless $H_{1.0}$ is directly connected to the localization length—a quantity with the physical meaning [16–18]). Assuming the χ^2_ν distribution for the $|c_i|^2$ components of a CS we obtain following formulae for generalized sum and entropy

$$S_e = N^{(1-e)/2}\left[\frac{2}{\nu}\right]^{e/2}\frac{\Gamma[(\nu+e)/2]}{\Gamma(\nu/2)} \tag{3.7}$$

and

$$H_u = N^{1-u}\left[\frac{2}{\nu}\right]^{u}\frac{\Gamma[(\nu+u)/2]}{\Gamma(\nu/2)}\left[\ln\left(N\frac{\nu}{2}\right)-\Psi\left(\frac{\nu}{2}+u\right)\right]. \tag{3.8}$$

The functions S_e and H_u also depend monotonically on the free parameters. The generalized sum $D_2 = S_e\sqrt{N}$ is presented in figure 1(*b*) as a function of e. The matrix dimension was taken $N = 100$. A broken vertical line is drawn at $e = 1.0$—the value of the parameter e used in the numerical study of kicked top. The function S_e loses its meaning if the parameter e tends to 2, since S_2 is equal to one for every normalized state. For $e > 2$, on the other hand, the generalized sum can also be useful. Quantity $S_{4.0}$, known as the inverse participation ratio [19-21], was exploited for many years in solid state physics to characterize the localization in a system.

4. Numerical study of kicked top

Periodically kicked tops have proven to be a suitable testing ground for various problems of quantum chaology. The dynamical variables of these models are the three components of an angular momentum operator $\boldsymbol{J}$ which obey the commutation relation $[J_i, J_j] = \mathrm{i}\varepsilon_{ijk}J_k$. The Hamiltonian

$$H = H_0 + V \sum_{n=-\infty}^{n=+\infty} \delta(t-n) \tag{4.1}$$

with H_0 and V being functions of J_i, $i = 1, 3$, describes the kicked top model. Depending on properties of H_0 and V systems pertaining to each of the three known universality classes exist [22]. The particular model defined by

$$H_0 = pJ_y \qquad V = k\frac{J_z^2}{2j} \tag{4.2}$$

under condition of classical chaos can be described by the orthogonal ensemble since an antiunitary symmetry exists [10]. For small values of the kicking strength k islands of stability appear in the classical phase space, and for k equal to zero the model becomes integrable.

It is convenient to analyse the quantum system in the eigenbasis of the operator J_z, $|j, m\rangle$, $m = -j, \ldots, j$. In the classical limit $j \to \infty$ the normalized vector $X = J/j$ lies on the unit sphere. The time evolution of the corresponding classical model can be given by a map for three components (X, Y, Z) of the angular momentum vector. The classical map reads [10]

$$\begin{aligned} X' &= (X\cos p + Z\sin p)\cos(kZ\cos p) + Y\sin(kX\sin p) \\ Y' &= -(X\cos p + Z\sin p)\sin(kX\sin p) + Y\cos(kZ\cos p) \\ Z' &= -X\sin p + Z\cos p. \end{aligned} \tag{4.3}$$

The above transformation maps the position of the top between subsequent kicks. Since the norm of vector X is conserved the dynamics of the system can be represented by a two-dimensional phase space. Two angles θ, φ have been chosen: $Z = \cos\theta$, $Y = \sin\theta\sin\varphi$.

Figure 2 shows several trajectories of the classical map (4.3) with $p = \pi/2$ and $k = 3.0$. The whole spherical surface is projected on a plane, so the lines $\theta = 0$ and $\theta = \pi$ correspond to the north and south pole, respectively. The value of the perturbation parameter k is chosen such that the chaotic layer dominates in the phase space, but four stability islands localized in the vicinity of fixed points are well distinguishable.

In order to compare the dynamics of the corresponding quantum model states localized in the phase space have to be introduced. We shall use directed angular momentum states $|\theta, \varphi\rangle$ generated from the state $|j, j\rangle$ by the unitary rotation operator

$$R(\theta, \varphi) = \exp[\mathrm{i}\theta(J_x\sin\varphi - J_y\cos\varphi)]. \tag{4.4}$$

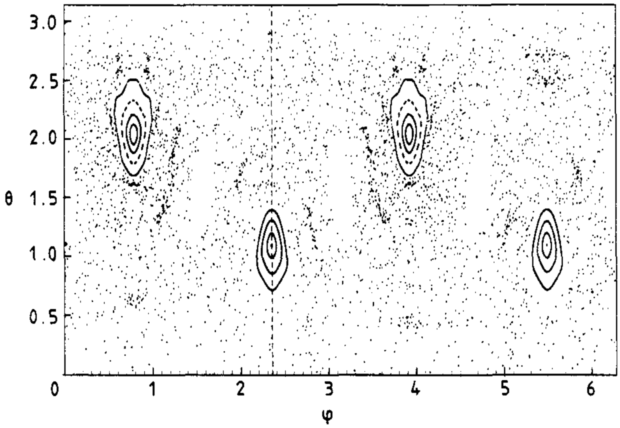


Figure 2. Phase space portrait of classical kicked top for $k=3.0$.

Such states exhaust the uncertainty principle with relative variance proportional to $1/j$ and shrinking to zero in the classical limit $j\to\infty$. The explicit expansion of the coherent state $|\theta, \varphi\rangle$ in the basis $|j, m\rangle$ reads [23]:

$$\langle\theta, \varphi|j, m\rangle=(1+\gamma\gamma^*)^{-j}\gamma^{j-m}\left[\binom{2j}{j-m}\right]^{1/2} \tag{4.5}$$

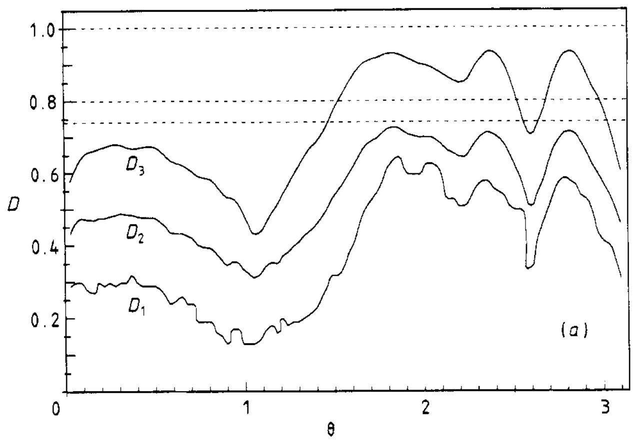
where $\gamma=\exp(\mathrm{i}\varphi)\tan(\theta/2)$.

We have constructed a family of CS localized on a meridian ($\varphi=3\pi/4$, $0\leqslant\theta\leqslant\pi$). This cross section of the phase space (denoted in figure 2 by a broken vertical line) intersects the stability island for $0.8\leqslant\theta\leqslant1.4$. The quasi-energy eigenstates were obtained by numerical diagonalization of the Floquet operator

$$F=\exp(-\mathrm{i}pJ_y)\exp(-\mathrm{i}kJ_z^2/2j) \tag{4.6}$$

and the CS were expanded in the eigenbasis of F. Three quantities defined in the previous paragraphs: the minimal number of relevant states M ($r=0.99$), the sum $S=S_{1,0}$ and the entropy $H_s=H_{1,0}$ were calculated for each coherent state. To present them on the same graph the following scaled quantities have been used: $D_1=M/N$, $D_2=S/\sqrt{N}$ and $D_3=[H_s+\Psi(3/2)]/\ln(N/2)$ (see (2.9)), where $N=2j+1$ is the matrix dimension.

The functional dependencies of D_i on θ are depicted in figure 3 for the same values of system parameters ($k=3.0$, $p=\pi/2$) with $j=50$ (figure 3(a)) and $j=200$ (figure 3(b)). Three horizontal broken lines are drawn at 0.73, 0.79 and 1.0 and denote predictions for D_1, D_2 and D_3 obtained in section 2 and 3 for fully chaotic systems described by χ_ν^2 distribution with $\nu=1$. At the first glance the presence of stability island in the classical phase space may be recognized—all three curves display a dominant minimum at $\theta\approx1.1$. Note the similar behaviour of all quantities also in the chaotic region of $\theta>1.8$. Also in this region each curve D_i lies below the corresponding broken line. According to equations (2.7), (2.9) and (3.4) all the quantities D_i do not depend on the size of matrix N. In the chaotic region of the classical phase space changes of the quantities investigated with the matrix dimension are insignificant indeed.



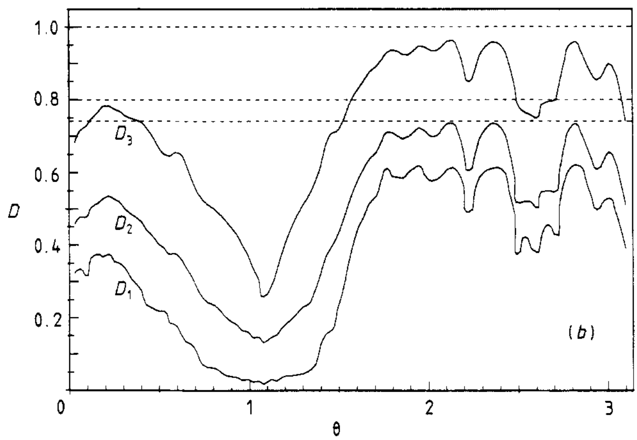


Figure 3. Properties of coherent states of the quantum system with $k = 3.0$ localized at the broken line of figure 2. Fraction of relevant eigenstates $D_1 = M/N$, scaled sum of moduli $D_2 = S/\sqrt{N}$ and scaled Shannon entropy $D_3 = [H_s + \Psi(3/2)]/\ln(N/2)$ are drawn as a function of θ. (*a*) $j = 50$. (*b*) $j = 200$.

It was suggested [10] that the correlation between the number of relevant eigenstates and the Lapunov exponent becomes stronger as j increases, since the CS of a smaller width becomes more precise tool to detect islands of stability. This statement is now supported by the following observation: the minimum of all curves is more pronounced in figure 3(*b*) obtained for a larger j value. This feature cannot be explained by means of χ^2_ν distribution, since the results obtained for the quantities D_i do not depend on the matrix dimension N. In other words, the χ^2_ν distribution is not fully adequate in the case of transition to classically regular motion.

We have shown how the analysed indicators can reveal local features of quantum dynamics. Now the possibility of distinguishing between systems pertaining to different universality classes will be demonstrated. Figure 4(*a*) is drawn for the system (4.2) with fully developed chaos ($k = 9.0$, $p = \pi/2$). The chaotic layer covers uniformly the whole phase space and all three curves D_i display only small fluctuations around the mean values. Predictions based on the χ^2_ν distribution are well fulfilled: all three indicators oscillate around the broken lines which denote values obtained for quantities D_i for the OE.

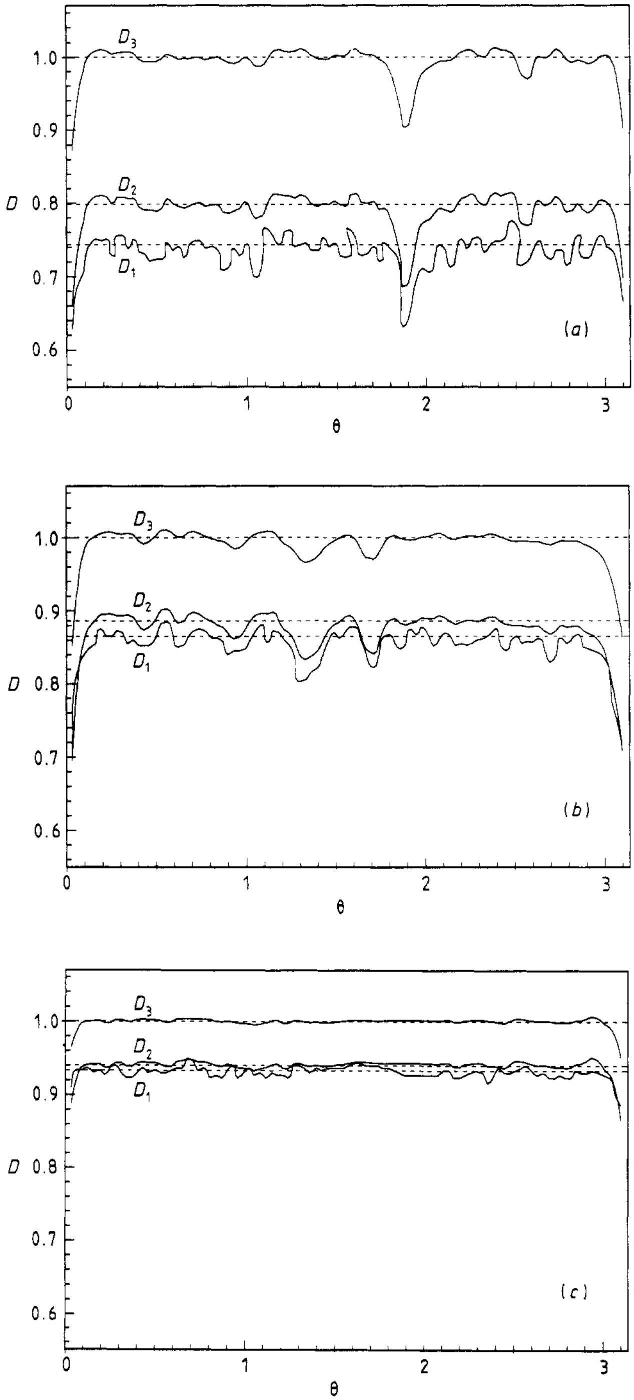


Figure 4. As in figure 3 for systems pertaining to different universality classes. (*a*) Orthogonal case $k = 9.0$, $d = 0.0$, $j = 200$. (*b*) Unitary case $k = 9.0$, $d = 1.0$, $j = 200$. (*c*) Symplectic case $p = k = 5.0$, $d_1 = 0.1$, $d_2 = 2.0$, $j = 299.5$.

Figure 4(*b*) shows an example of the model pertaining to the UE so the eigenvectors statistics is given by the χ^2_ν distribution with $\nu=2$. The Floquet operator consists of three unitary factors [10]

$$F=\exp(-\mathrm{i}pJ_y)\exp(-\mathrm{i}kJ_z^2/2j)\exp(-\mathrm{i}dJ_x^2/2j). \tag{4.7}$$

The diagonalization of (4.7) was performed for $p=\pi/2$, $k=9.0$ and $d=1.0$. The lowest broken line in figure 4(*b*) is plotted at $M_{0.99}(\nu=2)/N\approx 0.86$; the centre broken line plotted at $G(2)\approx 0.89$ denotes prediction for scaled sum D_2. The Shannon entropy is now scaled $D_3=[H_s+\Psi(2)]/\ln(N)$ so its value received from equation (2.9) is equal to one.

The system given by the Hamiltonian (4.2) does not pertain to the symplectic ensemble. The presumably simplest case reads [22]

$$H_0=pJ_z^2/j \qquad V=\frac{k}{j}(J_z^2+d_1[J_x,J_z]_+ + d_2[J_x,J_y]_+) \tag{4.8}$$

where $[\ ,\ ]_+$ denotes anticommutator. For a half-integer value of j the quartic level repulsion characteristic of the SE was found [22]. Figure 4(*c*) is obtained for the system (4.8) with $j=299.5$, $p=k=5.0$, $d_1=0.1$, $d_2=2.0$. For this half-integer value of j Kramers' degeneracy occurs; therefore each coherent state is built of $N=(2j+1)/2$ components. The Shannon entropy is now scaled as $D_3=[H_s+\Psi(3)]/\ln(2N)$, in order to have according to (2.9) $D_3=1$. Two other broken lines are placed at $G(4)$ and $M_{0.99}(4)/N\approx 0.933$. Also in the symplectic case all three indicators conform well to the predictions based on the χ^2_ν distribution.

The amplitudes of fluctuations of quantities D_i around the values predicted by equations (2.7), (2.9) and (3.4) are largest for OE (figure 4(*a*)), smaller for UE (figure 4(*b*)) and smallest for SE (figure 4(*c*)). Each component of orthogonal, unitary or symplectic ensembles is built of $\nu=1$, 2 or 4 independent variables, respectively. Calculation of values of every function of these components (like discussed indicators D_1, D_2, D_3) contains an intrinsic averaging over ν random variables. Thus smallest fluctuations are observed in the symplectic case with $\nu=4$.

5. Discussion

A possibility to obtain relevant information concerning local features of the quantum systems has been pointed out. There are several ways to analyse the whole set of expansion coefficients of a localized wavepacket in the basis of the Hamiltonian eigenstates. I have discussed three quantities revealing statistical properties of this expansion: the number of relevant eigenstates M, the sum of moduli of components S and the Shannon entropy H_s. All of them allow us to distinguish between quantum states placed in chaotic or regular regions of the classical phase space and may therefore be applied as quantum indicators of chaos.

It should be stressed, however, that the above criteria are not applicable without restriction: they are not appropriate for systems where the dynamical localization occurs, like the model of the kicked rotator [16, 24].

Each of the three chaos indicators may be generalized into a one-parameter family. Different members of each family are not distinguished from one another by any physical arguments. A family yields acceptable chaos indicators if its members follow one another reasonably smoothly as the parameter is varied continuously. All three families turn out to be equally acceptable in this sense. In particular, each of them allows the distinction of dominating regular and fully chaotic behaviour in the classical limit. Moreover, in the case of classical chaos the universality class of a given dynamics can be expressed as orthogonal, unitary or symplectic by each indicator.

Numerical studies of kicked tops confirm the predictions of random-matrix theory. The χ^2_ν distribution describes the statistics of components of a CS in chaotic systems, while the number of degrees of freedom ν is equal 1, 2 or 4, depending on the symmetry of a given model. On the other hand, use of the χ^2_ν distribution does not lead to appropriate dependence of the quantities M, S or H_s on the matrix dimension N in the mixed case of transition to regular motion. Therefore a need for a more adequate family of distributions, interpolating between the Porter–Thomas distribution (chaotic case) and $1/y$ behaviour (regular case) appears.

Of great interest is the question whether a direct correlation between the classical Lapunov exponent Λ and one of three quantities M, S or H_s describing the quantum system exists. Finding an explicit relation $M(\Lambda, N)$ would certainly push forward our understanding of the classical–quantum correspondence in the chaotic systems.

Acknowledgments

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Note added in proof. Quite recently W Wootters has pointed out that the statistics of components of coherent state in the eigenbasis of the system depends not only on the universality class of the system, but also on the symmetry properties of the CS itself (Wootters 1990 *Foundations of Physics* to appear). In particular the CS applied to investigate the 'orthogonal' top (4.6) are invariant under a generalized time-reversal symmetry [10]. Thus the values of all three indicators depicted in figure 4(*a*) agree with predictions obtained above for $\nu = 1$. On the other hand the eigenvector statistics of those CS which do not possess any antiunitary symmetry is well approximated by the case of $\nu = 2$, characteristic to the unitary ensemble, even though the system pertains to the orthogonal universality class (Życzkowski 1990 *Proc. Workshop on Quantum Chaos Trieste, 4.06-6.07.1990* (Singapore: World Scientific) to appear). I am indebted to W Wootters for drawing my attention to this point and providing his results prior to publication.

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