Universal Spectra of Random Lindblad Operators

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To understand the typical dynamics of an open quantum system in continuous time, we introduce an ensemble of random Lindblad operators, which generate completely positive Markovian evolution in the space of the density matrices. The spectral properties of these operators, including the shape of the eigenvalue distribution in the complex plane, are evaluated by using methods of free probabilities and explained with non-Hermitian random matrix models. We also demonstrate the universality of the spectral features. The notion of an ensemble of random generators of Markovian quantum evolution constitutes a step towards categorization of dissipative quantum chaos.

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Introduction.—Any real system is never completely isolated from its environment and the theory of open quantum systems [1–3] provides appropriate tools to deal with such phenomena as quantum dissipation and decoherence. In the Markovian regime (which assumes a weak interaction between the system and its environment and separation of system and environmental timescales), the evolution of an N-level open quantum system can be modeled by using the master equation $\dot{\rho}_t = \mathcal{L}(\rho_t)$. The corresponding Markovian generator $\mathcal{L}$ (often called a Lindblad operator or simply "Lindbladian" [1,4]) has the well-known Gorini-Kossakowski-Sudarshan-Lindblad form (GKSL) [5,6]

$$\mathcal{L}(\rho) = -i[H,\rho] + \mathcal{L}_D(\rho) = \mathcal{L}_U(\rho) + \mathcal{L}_D(\rho), \quad (1)$$

with the dissipative part

$$\mathcal{L}_D(\rho) = \sum_{m,n=1}^{N^2-1} K_{mn}\left[F_n\rho F_m^\dagger - \frac{1}{2}(F_m^\dagger F_n\rho + \rho F_m^\dagger F_n)\right], \quad (2)$$

where traceless matrices $\{F_n\}$, $n = 1,2,3,\ldots,N^2-1$, satisfy the orthonormality $\mathrm{Tr}(F_n F_m^\dagger) = \delta_{n,m}$. Finally, the complex "Kossakowski matrix" $K = \{K_{mn}\}$ is positive semidefinite. The solution of the master equation $\dot{\rho}_t = \mathcal{L}(\rho_t)$ gives rise to the celebrated Markovian semigroup $\Lambda_t = e^{t\mathcal{L}}$, such that for any $t \geq 0$ map Λ_t represents a quantum channel, a completely positive and trace-preserving linear map [7].

In this Letter, we analyze the spectral properties of random Lindblad operators. Spectral analysis lies in the heart of quantum physics. In the static case, the spectrum of the Hamiltonian provides full information about the possible state of the system and system evolution. Such an analysis plays also a key role in the study of dissipative quantum evolution—eigenvalues and eigenvectors of the Lindblad operator provide the full information about the dynamical properties of the open system [9]. Spectra of dynamical maps were recently addressed in Ref. [10] in connection to quantum non-Markovian evolution. This connection was experimentally verified recently [11], which proves that spectral techniques can be used to characterize non-Markovian behavior as well. Here, instead of analyzing specific physical models (like in Refs. [10,11]), we look for universal spectral properties displayed by "typical" Lindblad operators. It should be stressed that the standard examples of generators of order $N = 2$, usually considered in the literature, do not display universal features. We address the problem in the limit $N \gg 1$ by using the apparatus of the random matrix theory (RMT) [12].

The RMT has already found many applications in physics. It started from the Wigner statistical approach to nuclear physics [13] and a series of Dyson papers on statistical theory of spectra [14–17]. It was soon realized that quantum dynamics corresponding to classically chaotic dynamics can be described by suitable ensembles of random matrices [18–20]. Depending on the symmetry properties of the system investigated, one may use orthogonal, unitary, or symplectic ensembles [12]. Similar ideas found applications in disorder systems and single-particle [21] and many-body [22] ones. From a different perspective, a deep connection to RMT was observed in

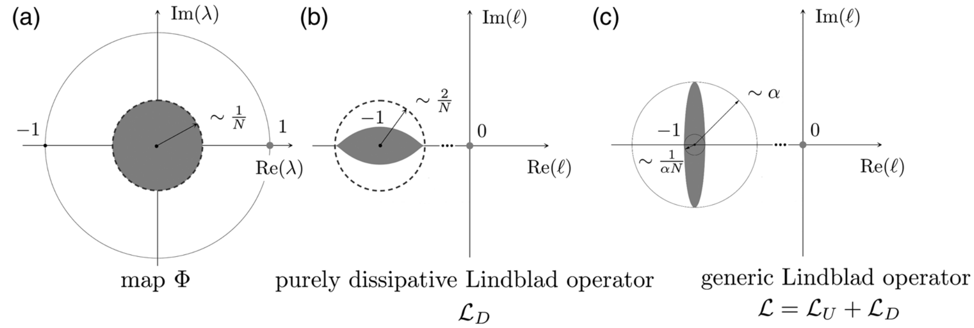

FIG. 1. Eigenvalue localization areas (gray) of a (a) random quantum channel, (b) random purely dissipative Lindblad operator, Eq. (3), and (c) random generic Lindbladian with relative weight of the unitary component α, Eq. (8). While for the random channel (a) the distribution approaches a Girko's disk upon the increase of the number of system levels N [27], eigenvalues of the random purely dissipative Lindblad operator (b) fill the interior of the universal lemonlike contour, Eq. (7). For the Lindblad operator with the unitary component (c), the spectral boundary approaches an ellipse upon increase of α.

the models of 2D quantum gravity [23,24] and gauge theories [25].

In the case of discrete dynamics described by quantum operations, various ensembles of random channels are known [26,27]. Recently, the RMT found interesting applications in quantum information theory [28–30]. In the case of continuous quantum dynamics, a class of Lindblad equations with decay rates obtained by tracing out a random reservoir was studied in Refs. [31,32]. RMT was applied to open quantum systems in the context of scattering matrices and non-Hermitian effective Hamiltonians [33].

Our perspective in this paper is different: We introduce an ensemble of random Lindblad operators, which describe continuous time evolution of an N-level open quantum system, and evaluate universal properties of operator spectra. Namely, we analyze the distribution of eigenvalues of a randomly chosen operator $\mathcal{L}$ and study the scaling of spectral characteristics with N. For generators of classical Markovian evolution, a similar program was initiated and realized by Timm in Ref. [34].

We start by briefly recalling main results concerning random quantum channels [27,35]. Next, we analyze the extreme case of purely dissipative evolution, $H = 0$ and $\mathcal{L} = \mathcal{L}_D$. This part can be considered as a quantum extension of the program outlined in Ref. [34] [from the opposite perspective, classical Pauli rate equations can be obtained as a reduction of the quantum master equation (1)]. Finally, we address the general situation, when both unitary and dissipative components $\mathcal{L}_U$ and $\mathcal{L}_D$ are present.

Random quantum channels.—An ensemble of random channels (i.e., completely positive and trace-preserving transformations [1–3]) $\Phi: M_N(\mathbb{C}) \to M_N(\mathbb{C})$ can be defined by the flat Hilbert-Schmidt measure in the space of all quantum operations. The spectrum of Φ includes the leading eigenvalue $\lambda_1 = 1$, corresponding to the invariant state, while all remaining eigenvalues fill a disk of radius $R = 1/N$ centered at zero; see Fig. 1(a). The bulk of the spectrum can be obtained by sampling random matrices $(1/N)G_R$ [27], with G_R being a member of a real Ginibre ensemble [36–39].

Thus, for a generic superoperator Φ the size of its spectral gap, $\Delta_N = \lambda_1 - |\lambda_2| = 1 - 1/N$, increases with dimension N, so the convergence to an invariant state becomes exponentially fast. For a large N, a typical channel becomes close to a one-step contraction, which sends any initial state into the invariant state, $\Phi(\rho) = \rho_{\text{inv}} = \Phi(\rho_{\text{inv}})$. It is known [40] that a typical channel is close to a unital one and the correction term $\Phi(\mathbb{1}) - \mathbb{1}$ behaves like a random Hermitian matrix of the Gaussian unitary ensemble with an asymptotically vanishing norm.

Purely dissipative random Lindblad operators.—To generate a random operator $\mathcal{L}_D$, we fix an orthonormal Hilbert-Schmidt basis $\{F_n\}$ [41] and first sample a random Kossakowski matrix K. There are many ways to do such sampling. However, as we show below, a particular way in which this non-negative order $N^2 - 1$ matrix is sampled is not important: The spectral features of random purely dissipative Lindbladians are *universal*.

The most intuitive way is to sample K from the ensemble of square complex Wishart matrices [50]. A Wishart matrix [51] has the structure $W = GG^\dagger \geq 0$, where G is a complex square Ginibre matrix with independent complex Gaussian entries [52]. Such a choice is also physically motivated by the fact that these ensembles of random matrices correspond to nonunitary evolution of quantum dynamical systems under the assumption of classical chaos [18,35].

We use the following normalization condition $\mathrm{Tr}K = N$, that is, $K = NGG^\dagger/\mathrm{Tr}GG^\dagger$. Note that eigenvalues of K, γ_m, $m = 1, \ldots, N^2 - 1$, which can be interpreted as decay rates [1], are distributed according to the universal Marchenko-Pastur law [51] with the mean value $\langle\gamma\rangle \sim 1/N$. By diagonalizing the Kossakowski matrix, one can reduce the form of $\mathcal{L}_D$ to

$$\mathcal{L}_D(\rho) = \sum_{m=1}^{N^2-1} \gamma_m \left[V_m \rho V_m^\dagger - \frac{1}{2}(V_m^\dagger V_m \rho + \rho V_m^\dagger V_m) \right], \quad (3)$$

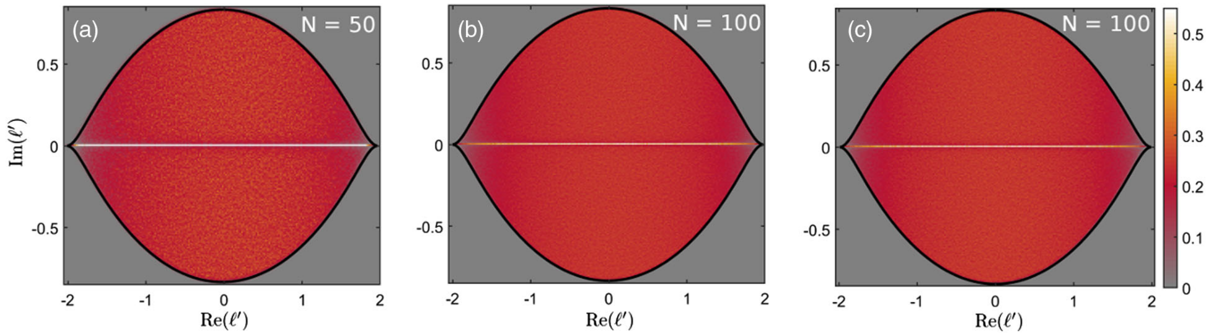

FIG. 2. Spectral density $P[\mathrm{Re}(\ell'), \mathrm{Im}(\ell')]$ of the rescaled eigenvalues, $\ell' = N(\ell + 1)$, from the spectrum of random purely dissipative Lindblad operators $\mathcal{L}_D$ for $N = 50$ and 100. We use two different sampling procedures for $N = 100$ (a),(b), sampling the Kossakowski matrix from the Wishart ensemble and (c) an alternative procedure (Supplemental Material [41]). Note a perfect agreement with the asymptotic boundary of the spectral bulk, Eq. (7) (thick black line), derived with the random matrix model (6). Observe also a concentration of eigenvalues along the real axis, accompanied by depletion nearby—compare to Fig. 3(a)—which decreases with N. Each distribution was sampled with 10^3 realizations.

where V_m are called "jump operators" [53]. $\Phi(\rho) = \sum_m \gamma_m V_m \rho V_m^\dagger$ defines a Kraus representation of a completely positive map. Moreover, $\sum_m \gamma_m V_m^\dagger V_m = \Phi^\dagger(\mathbb{1})$, where $\Phi^\dagger$ is the dual map, $\mathrm{Tr}[A \cdot \Phi^\dagger(B)] = \mathrm{Tr}[\Phi(A) \cdot B]$, and $\mathbb{1}$ is the identity matrix in $M_N(\mathbb{C})$. Therefore, Eq. (3) can be rewritten as

$$\mathcal{L}_D(\rho) = \Phi(\rho) - \frac{1}{2}[(\Phi^\dagger(\mathbb{1})\rho + \rho\Phi^\dagger(\mathbb{1})], \tag{4}$$

which shows that the purely dissipative Lindblad generator is fully determined by a completely positive map Φ. If, in addition, Φ is trace preserving, i.e., it is a quantum channel, we have $\mathcal{L}(\rho) = \Phi(\rho) - \rho$. This is not the case in general, and the Hermitian translation matrix $X = \Phi^\dagger(\mathbb{1}) - \mathbb{1}$ does not vanish. Making use of this notation, we rewrite the Lindblad operator as $\mathcal{L}_D(\rho) = [\Phi(\rho) - \rho] - \frac{1}{2}(X\rho + \rho X)$.

Due to the trace-preserving quantum Markovian dynamics, a Lindblad generator always has a zero eigenvalue. If Φ is a quantum channel, the spectrum of $\mathcal{L}_D$ is the spectrum of Φ shifted by -1. Thus, the leading eigenvalue $\lambda_1 = 1$ is translated into $\ell_1 = 0$ and the Girko disk is now centered at $z = -1$.

To sample spectra of random Lindbladians, we generate 10^3 realizations for different values of N, ranging from 30 to 100. In order to reveal the universality of spectra of the operators, it is useful to apply an affine transformation, $\mathcal{L}_D' = N(\mathcal{L}_D + 1)$ [41]. Then the bulk of the spectrum of $\mathcal{L}'$ becomes scale invariant and independent of N, see Figs. 2 and 3(a).

Sampled eigenvalue distributions $P[\mathrm{Re}(\ell'), \mathrm{Im}(\ell')]$ are significantly different from the Girko disk and display a universal lemonlike shape. Already for $N = 50$, a *single* realization is enough to reproduce the universal shape, see Fig. 3(b). From the scale invariance, it follows that the spectral gap of $\mathcal{L}_D$ scales as $\Delta_N \simeq 1 - (2/N)$. It is clear that the very term $(X\rho + \rho X)$ is responsible for the "disk $\to$ lemon" deformation. The density inside the lemon is manifestly nonuniform. Also notable is the eigenvalue concentration along the real axis and the corresponding depletion nearby, see Fig 3(a). Although $\mathcal{L}_D$ is represented by a complex matrix, it can be made real by a similarity transformation, which explains the concentration [37,39,55].

Finally, we performed sampling by using alternative algorithms (see Supplemental Material [41]) and obtained near identical results (differences are within the sampling errors) for $N \geq 50$; see Fig. 2(c). This confirms the universality of the spectral distribution. It is noteworthy that a similar-looking shape of the eigenvalue distribution was observed with classical random transition rate matrices [see Fig. 7(b) in Ref. [34]]. However, the singularities at the real poles are much stronger in the classical case [56].

Random matrix model.—Let us recall that the spectrum of $\mathcal{L}_D$ represented in (4) coincides with the spectrum of the following $N^2 \times N^2$ complex matrix $\mathcal{L}_{mn} = \mathrm{Tr}[F_m \mathcal{L}_D(F_n)]$.

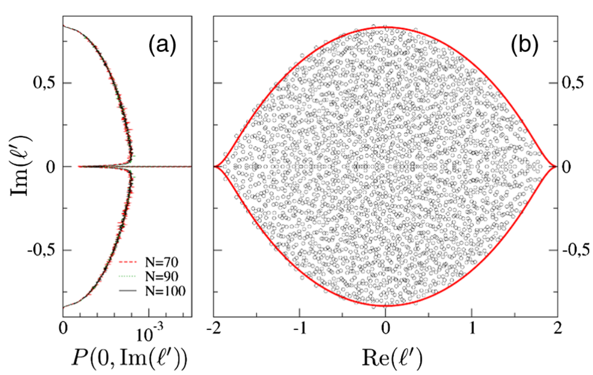

FIG. 3. (a) Marginal distribution, $\mathrm{Re}(\ell') = 0$, of rescaled eigenvalues for three different values of N. (b) Rescaled eigenvalues ℓ' (empty dots) of a single Lindblad operator realization for $N = 50$. Red outer contour is the boundary derived from the random matrix model, Eq. (6).

This matrix becomes real if basis matrices F_m are Hermitian, due to the fact that $\mathcal{L}_D$ is Hermiticity preserving. Another well-known matrix representation of the Lindblad operator reads

$$\hat{\mathcal{L}}_D = \hat{\Phi} - \mathbb{1} \otimes \mathbb{1} - \frac{1}{2}(X \otimes \mathbb{1} + \mathbb{1} \otimes \bar{X}), \tag{5}$$

where $\hat{\Phi} = \sum_{m=1}^{N^2-1} \gamma_m V_m \otimes \bar{V}_m$, and $\bar{V}_m$ stands for the complex conjugation. Note that $\hat{\Phi}$ is neither Hermitian nor real; however, the term $X \otimes \mathbb{1} + \mathbb{1} \otimes \bar{X}$ is perfectly Hermitian. To understand the spectrum of $\mathcal{L}'_D$, we use the matrix representation (5) and approximate its rescaled version with the following random matrix (RM) model:

$$\hat{\mathcal{L}}_D{}' \approx G_R - (C \otimes \mathbb{1} + \mathbb{1} \otimes C). \tag{6}$$

The $N^2 \times N^2$ matrix G_R is sampled from the real Ginibre ensemble, while C approximates X by a symmetric $N \times N$ Gaussian orthogonal ensemble (GOE) matrix [40].

Matrices are normalized as $\mathrm{Tr} G_R G_R^\dagger = N^2$, so that its spectrum covers uniformly a disk of radius 1, while $\mathrm{Tr} C^2 = N/4$ assures that its density forms the Wigner semicircle of radius 1. The scaling and parameters of the model follow from the normalization of the Kossakowski matrix (Supplemental Material [41]).

We approach spectral properties of the RM model (6) with the quaternionic extension of free probability [57–61]. Within this framework, we determine the border of the spectrum of $\mathcal{L}'_D$ as given by the solution of the following equation involving a complex variable z [41]:

$$\mathrm{Im}[z + G(z)] = 0, \tag{7}$$

with

$$G(z) = 2z - \frac{2z}{3\pi}\left[(4+z^2)E\left(\frac{4}{z^2}\right) + (4-z^2)K\left(\frac{4}{z^2}\right)\right],$$

where $E(k)$ and $K(k)$ are complete elliptic integrals of the first and second kind, respectively. The results of the sampling are in perfect agreement with this border, see Figs. 2 and 3. Evaluation of the spectral density inside the "lemon" is a much harder task; it could potentially be performed with diagrammatic techniques [58].

General case of random Lindbladians.—Finally, we include the unitary component $\mathcal{L}_U$ into the Lindblad operator $\mathcal{L}$. For that we use random Hamiltonian H, which is sampled from the Gaussian unitary ensemble (GUE). To compare the spectra of the general and the purely dissipative Lindblad operators, we normalize the Hamiltonian, $\mathrm{Tr} H^2 = 1/N$, and introduce a parameter $\alpha \geq 0$, which weights contribution of the unitary component. The corresponding Lindbladians can be written as [see Eq. (4)]

$$\mathcal{L}(\rho) = -\frac{i\alpha}{\hbar}(H\rho - \rho H) + \Phi(\rho) - \frac{1}{2}[\Phi^\dagger(\mathbb{1})\rho + \rho\Phi^\dagger(\mathbb{1})]. \tag{8}$$

The sampled spectra of the operator $\mathcal{L}' = N(\mathcal{L} + 1)$, for different values of α, are shown in Figs. 4(a), 4(b), and 4(d) (see also Supplemental Material [41]). Similar to the case of purely dissipative evolution, we find a perfect scale invariance starting from $N \geq 50$.

To capture the shape of the spectra with the RMT, we transform expression (8) into

$$\hat{\mathcal{L}} = \hat{\Phi} - \mathbb{1} \otimes \mathbb{1} - \left(\frac{1}{2}X + i\alpha H\right) \otimes \mathbb{1} + \mathbb{1} \otimes \left(\frac{1}{2}\bar{X} - i\alpha\bar{H}\right). \tag{9}$$

The spectrum of $\hat{\mathcal{L}}$ can be explained by updating the matrix model (6),

$$\hat{\mathcal{L}}' \approx G_R - (W \otimes \mathbb{1} + \mathbb{1} \otimes \bar{W}), \tag{10}$$

where G_R is again taken from the real Ginibre ensemble, while the extended correction term $W = C + i\alpha H'$ contains

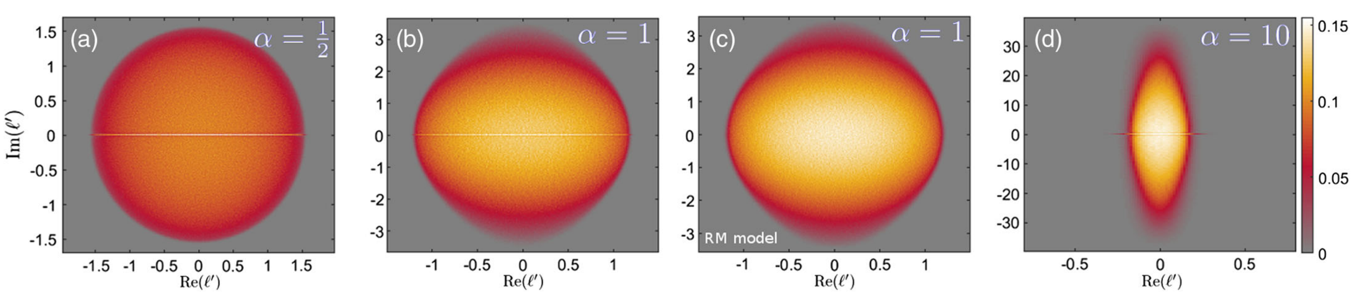

FIG. 4. Probability distributions $P[\mathrm{Re}(\ell'), \mathrm{Im}(\ell')]$ of the rescaled eigenvalues, $\ell' = N(\ell + 1)$, from the spectrum of random Lindblad operators $\mathcal{L}$, Eq. (8), for $N = 100$ and different values of the unitary component weight α. (a),(b),(d) We present here the results of the sampling of the Kossakowski matrix from the Wishart ensemble, emphasizing that alternative generation procedures (Supplemental Material [41]) yield the same results. (c) Spectral distribution obtained with the random matrix model, Eq. (10). Each distribution was sampled with 10^3 realizations. Additional normalization of the densities is performed in order to keep maximal values of all distributions equal.

now a random GOE matrix C and an anti-Hermitian term proportional to a GUE matrix H' normalized as $\mathrm{Tr}H'^2 = N$. Spectral density of the RMT model for $\alpha = 1$ is shown in Fig. 4(c). It reproduces spectral density of the corresponding Lindbladian ensemble (except of eigenvalue concentration at the real axis [37,39,55]).

Eigenvalues of W uniformly cover an ellipse with semiaxes $1/\sqrt{1+4\alpha^2}$ and $4\alpha^2/\sqrt{1+4\alpha^2}$. Spectral density of $\hat{\mathcal{L}}'$ is therefore a (classical) convolution of two uniform densities supported on these ellipses followed by free convolution with the Girko disk of unit radius; see Fig. 1(c). Contrary to the case of purely dissipative Lindbladians, it is not possible to find an analytically spectral boundary in this case. However, when $\alpha = \frac{1}{2}$, it immediately follows (since a convolution of two disks is a disk) that the spectral boundary is a circle. This is in full agreement with the results of the sampling; see Fig. 4(a).

Conclusions.—Universal spectral features of different ensembles of unitary evolution generators—that are Hamiltonians—are the main pillar of the quantum chaos (QC) theory [18]. A notion of an ensemble of random operators of quantum Markovian evolution is therefore a first step in generalization of QC to open quantum systems. Two next steps would be (i) establishing links between the idea of "typical Lindbladian" and physical models [62] and (ii) evaluation of spectral properties of steady states of random Lindbladians.

Finally, our approach works equally well in the classical limit, where continuous dynamics in the space of probability distributions is determined by random transition rate matrices. By using a free probabilities apparatus, we were able to derive analytically [56] the boundary of the eigenvalue distributions reported earlier in Ref. [34].

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Note added.—One of the authors (W. T.) attended a talk by Tankut Can given at the conference in Yad Hashmona (Israel), in which a parallel project on random Lindblad operators was presented. Recently, three other papers on related subjects have appeared [65–67].

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Correction: An incorrect affiliation for the fourth author was processed and has been fixed.