Hadamard2020+2

Book of Abstracts

June 27 – July 2, 2022

https://chaos.if.uj.edu.pl/hadamard2020/

Talks and Abstracts

In order of appearance in the Programme of the Workshop:

Circulant Complex Hadamard Matrices of Order 10

June 27
9:30

Markus Grassl

International Centre for Theory of Quantum Technologies (ICTQT), University of Gdańsk, Jana Bażyńskiego 1A, 80-309 Gdańsk, Poland

We discuss results on circulant complex Hadamard matrices of order N obtained from solutions to the cyclic N-th root problem. Together with the Fourier matrix and identity, those yield triples of mutually unbiased bases (MUBs). We focus on dimension 10 and present a complete set of algebraic solutions to the cyclic-10 problem.

The talk is based on joint work with Ferenc Szöllősi.

Factors of Four

June 27
10:50

Ingemar Bengtsson

Department of Physics, Stockholm University, SE-106 91 Stockholm, Sweden

Four is the lowest dimension in which Hadamard matrices, and SIC-POVMs, become non-trivial, and also the lowest dimension in which non-isomorphic Heisenberg groups appear. I will discuss some of its less well known properties from that point of view. I will also discuss some special properties that are inherited by higher dimensions that are divisible by four.

June 27
11:40
online talk

Geometric Properties of SIC-POVM Tensor Square

Danylo Yakymenko

Institute of Mathematics, National Academy of Sciences of Ukraine, 01024 Ukraine, Kyiv-4, Tereschenkivska str., 3

It is known that if d^2 vectors in a d-dimensional Hilbert space H form a symmetric, informationally complete, positive operator-valued measure (SIC-POVM), then the tensor squares of these vectors form an equiangular tight frame in the symmetric subspace of $H \otimes H$. It's proved that, for any SIC-POVM of the Weyl–Heisenberg group covariant type (WH-type), this frame can be obtained by projecting a WH-type basis in $H \otimes H$ onto the symmetric subspace. We give a full description of the set of all WH-type bases, so that this set could be used as a search space for finding SIC solutions. Also, we show that a particular element of this set is close to a SIC solution in some structural sense. Finally, we give a geometric construction of SIC-related symmetric tight fusion frames that were discovered in odd dimensions in Appleby et al. (J Phys A 52(29):295301, 2019).

The talk is based on the joint work with Vasyl Ostrovskyi.

June 27
14:30

Hilbert's Twelfth Problem and the Geometry of Quantum State Space

Marcus Appleby

Centre for Engineered Quantum Systems, School of Physics, University of Sydney, Sydney NSW 2006, Australia

We demonstrate an intricate (though still conjectural) interplay between the properties of an abstract number field and the geometry of quantum state space. The work builds on results previously obtained by Kopp for a certain subset of SICs (symmetric informationally complete measurements) in the sequence of prime dimensions equal to 5 mod 6, and by Appleby, Bengtsson, Grassl, Harrison and McConnell for a certain subset of prime dimensions equal to a square plus 3. These previous works presented evidence that Stark units play a fundamental role in the construction of the SICs considered. We present strong evidence that Stark units play a fundamental role in every Weyl-Heisenberg covariant SIC in every dimension. We also present evidence that such properties as number of unitarily inequivalent orbits of SICs in each dimension, and the order of the symmetry group, follow directly from the underlying number theory.

This is a joint work with Steven Flammia and Gene Kopp.

Mutually Unbiased Measurements and Hadamard Matrices of Unitaries

June 27
15:20

Máté Farkas

ICFO-Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels, Spain

It is well-known that mutually unbiased bases (MUBs) correspond to complex Hadamard matrices that specify the phases of the overlaps between the vectors from the two bases. It is also well-known that MUBs correspond to quantum measurements that are complementary: if measuring a state in a basis yields a definite outcome, then measuring the same state in a basis unbiased to the first one yields a uniformly random outcome. In this work, we generalise this latter definition to a more operational setting, removing the assumption that the dimension of the Hilbert space equals the number of measurement outcomes. We call measurement pairs satisfying this relaxed complementarity definition mutually unbiased measurements (MUMs). MUMs have a simple algebraic characterisation in terms of the measurement operators. Furthermore, they can also be characterised by what we call a Hadamard matrix of unitaries: a block matrix whose every entry is a unitary matrix, satisfying orthogonality conditions analogous to complex Hadamard matrices. This characterisation allows us to derive a simple condition for a pair of MUMs to correspond to a direct sum of MUBs, based on the commutation relations of the unitary entries. Furthermore, through a group homomorphism we prove a correspondence of quaternionic Hadamard matrices to Hadamard matrices of unitaries with block size two. This correspondence allows us to construct MUM pairs that are not direct sums of MUBs for small prime outcome numbers, using known quaternionic constructions. We can further show that—in stark contrast with MUBs—there exists an unbounded number of MUMs for every outcome number. Last, we discuss the connection of MUMs to quantum information, namely to Bell inequalities (also in relation to the MUB problem), superdense coding, entropic uncertainty relations, measurement incompatibility, and steering inequalities.

Based on: Science Advances 7 eabc3847, `arXiv:2204.11886`, `arXiv:2203.09429`.

June 28
9:30

Hadamard Matrices and Continuous Variables

Stefan Weigert

Department of Mathematics, University of York, York YO10 5DD, United Kingdom

In finite-dimensional quantum systems, a close link exists between Hadamard matrices and mutually unbiased bases. I will discuss an analogous relation for so-called "continuous variables" i.e. for quantum systems described in terms of an infinite-dimensional Hilbert space. In this setting, Hadamard matrices with "continuous" labels emerge naturally. Some explicit examples will be presented.

This is a joint work with M. Wilkinson and A. Beales.

June 28
10:50

2×2 Submatrices of Hadamard Matrices

Robert Craigen

Department of Mathematics, University of Manitoba, Winnipeg, MB R3T 2N2, Canada

What does a Hadamard matrix look like at the smallest nontrivial level—2×2—of submatrix structure? We explore this question both empirically and analytically, obtaining some perhaps surprising answers, focussing on the rank of 2×2 submatrices and their distribution under partitions. Obviously every rank two submatrix will itself be a Hadamard matrix.

The behavior of small Hadamard matrices suggests provisional conjectures:

1. Every Hadamard matrix of order > 2 can be partitioned into 2×2 rank one submatrices; and

2. Every Hadamard matrix of order > 1 can be partitioned into 2×2 rank two submatrices.

One of these is resolved; the other remains open. Some ramifications are explored.

Complex Hadamard Matrices Versus Uncertainty, Incompatibility and Nonclassicality in Quantum Mechanics

June 28
11:40

Stephan De Bièvre

Département de Mathématiques - Lab. P. Painlevé, Bâtiment M2, Université de Lille, 59655 Villeneuve d'Acq

In this talk, I will report on recent results on the link between complex Hadamard matrices and the notions of incompatibility, uncertainty and nonclassicality in quantum mechanics. Given an arbitrary unitary d by d matrix U, one can prove a very simple and general multiplicative support uncertainty relation that extends the well known Donoho-Stark uncertainty relation proven in 1989. In 2005, T. Tao established that a much improved additive support uncertainty principle is valid for DFT matrices, but only in prime dimension. We have shown this latter uncertainty relation has a natural interpretation in quantum mechanics in terms of a strong notion of incompatibility of observables that we refer to as complete incompatibility. In other words, the DFT is completely incompatible only in prime dimension. This raises a number of questions. We have for example shown that the completely incompatible U form an open dense set, but it is not clear in which dimensions there exist completely incompatible complex Hadamard matrices: in dimension $d \leq 5$, the question is easily settled, but not in higher dimensions. Furthermore Kirkwood (1935) and, independently, Dirac (1945) showed how, given a unitary matrix U, one can associate to each vector in $\mathbb{C}^d$ a natural complex quasi-probability distribution akin to the Wigner function. A vector is said to be KD-classical when this distribution is nonnegative, and hence a probability distribution. We will present necessary conditions for vectors to be classical and discuss to what extent they are sharp for (completely incompatible) Hadamard matrices in particular.

The Hadamard Maximal Determinant Problem

June 28
14:30

Padraig Ó Catháin

Fiontar agus Scoil na Gaeilge, Dublin City University

Hadamard's famous paper of 1893 discusses complex matrices which meet his bound with equality. Slightly less well known is the body of work on matrices with entries in $\{1, -1\}$ with maximal determinant, when the dimension is not a multiple of 4. I will survey the main techniques, bounds and constructions to be found in the literature.

While the analogous complex Hadamard matrices (particularly with k^{th} roots as entries) have been well studied, much less is known about complex maximal determinant matrices (over a fixed finite extension of the rationals) when the bound is not attained. I will present some open questions and directions for future research.

This is joint work with Patrick Browne, Ronan Egan, Fintan Hegarty and Guillermo Nuñez Ponasso.

June 28
15:20

The Maximal Determinant Problem over Cyclotomic Fields

Guillermo Nuñez Ponasso

Worcester Polytechnic Institute, 100 Institute Rd, 01609, Worcester, Massachusetts, USA

We discuss generalisations of the Maximal Determinant Problem, focusing on the cases of the third and fourth roots of unity.

June 28
16:20

A Spectral Sequence for Cohomology Developed Matrices and Tensors

Giora Dula

Netanya Academic College, Netanya, Israel

Given a finite group G, a cocyclic matrix based on G can be constructed based on the data of a cocycle $\omega \in H^2(G, \mu)$ and an arbitrary function $f : G \mapsto \mu$ where μ is a multiplicative subgroup of complex numbers. Many new Hadamard and weighing matrices were found by this method, and many known others proved to be cocyclic. We propose, for a finite group G, to consider finite G sets X and Y, and cohomology classes in $H^i(G, -), i = 0, 1, 2$ for certain G modules related to X and Y, in order to construct experimentally Hadamard and weighing matrices. In the particular case $X = Y = G$ (action given by translation), the class in H^1 in question vanishes and we recover the known cocyclic construction. Our construction has been employed to find an infinite family of weighing matrices. Allowing the group G to act on the matrix coefficients (e.g. allowing extra "Hadamard" operations like complex conjugation and other Galois transformations), two infinite families of Hadamard matrices were found. All those 3 families are probably new. There are few further generalizations of this construction, one of which is to replace matrices with tensors of any dimension. Spectral sequences play a main role in this study.

This is a joint work with Assaf Goldberger.

June 28
16:55

Combinatorial Designs in the Study of Classical and Quantum Cellular Automata

Balázs Pozsgay

Eötvös Loránd University, Budapest, Egyetem tér 1-3, 1053

Combinatorial designs recently made a new appearance in theoretical physics, namely in the study of non-equilibrium dynamics of one dimensional models. The relevant mathematical objects are the so-called dual unitary or bi-unitary matrices, which can be constructed for example from Hadamard matrices or from "broken" orthogonal arrays, where some of the orthogonality conditions are relaxed. In the talk we explain the relevance of this topic for theoretical physics, and we explain the construction of the cellular automata which use the combinatorial designs. Afterwards we mention some recent results of the authors, together with a list of open problems.

This is a joint work with Márton Borsi.

Rigidity and Characterization Results Regarding Complete Systems of MUBs

June 29
9:30

Mihály Weiner

Budapest University of Technology and Economics (BME), Department of Analysis,
H-1111 Budapest Műegyetem rkp. 3–9 Hungary

In my talk I will summarize results from two papers: one from 2020, joint with my students S. Nietert and Sz. Zsombor, and a more recent one which is joint with M. Matolcsi.

What do we mean by "rigidity"? In mathematics this term is used when an object is completely determined by a proper subset of its points. (E.g. a square has some rigidity, since knowing just 3 of its vertices allows the determination of its 4^{th} vertex.) Regarding complete systems of MUBs, one may ask: how rigid is such a collection? That is, in general, at least how many bases need to be fixed to fully determine (up to ordering and multiplication by phase-factors) the entire collection? We now have a precise answer to this question.

Although a quite different question, but we can also consider how rigid is the definition of complete system of MUBs. In order for a collection of $n(n+1)$ unit vectors in an n-dimensional space to form such a system, one needs to be able to arrange them in $n+1$ orthonormal, pairwise mutually unbiased bases. However, can we loosen the definition and just require those $n(n+1)$ unit-vectors to be such, that the absolute value of the scalar product of any two of them is either $1/\sqrt{n}$ or zero? Apart from answering, I will also explain the motivation of this problem.

The Finite Fourier Transform and Projective 2-Designs (Topics in Quantum Design Theory)

June 29
10:50
online talk

Gerhard Zauner

http://www.gerhardzauner.at/

Weyl-Heisenberg matrices and the Clifford group in complex vector spaces are revisited. Some new matrix algebras and related Quantum Designs are presented.

June 29
11:40

MUB-Triplets and Hadamard Cubes

Máté Matolcsi

Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences, Reáltanoda u. 13-15, H-1053 Budapest, Hungary

A pair of mutually unbiased bases (MUBs) correspond to a complex Hadamard matrix in a natural way. In this talk we consider a triplet of MUBs, and show that it leads to an $n \times n \times n$ object, which we call a “Hadamard cube”. Namely, each cross-section of a Hadamard cube is a complex Hadamard matrix. We will observe some further properties of this cube, and contemplate whether such cubes exist with real Hadamard matrices.

Joint work with M. Weiner, D. Varga, A. Matszangosz.

June 29
14:30

Generalized Partially Bent Functions, Generalized Perfect Arrays and Cocyclic Butson Matrices

José Andrés Armario Sampalo

Universidad de Sevilla, Sevilla, Spain

Equivalences between generalized bent functions, group invariant (also known as group-developed) Butson Hadamard matrices, and abelian splitting relative difference sets are known. In this talk we will discuss about a broader network of equivalences, including several extra objects, by considering Butson matrices that are cocyclic rather than strictly group invariant. We will show some examples.

This is a joing work with Dane Flannery and Ronan Egan.

June 29
15:20

Constructing Cocyclic Hadamard Matrices of Order $4p$

Santiago Barrera Acevedo

School of Mathematics, Monash University, Clayton 3800, Victoria, Australia

Cocyclic Hadamard matrices (CHMs) were introduced by de Launey and Horadam as a class of Hadamard matrices with interesting algebraic properties. Then Ó Catháin and Röder described a classification algorithm for CHMs of order 4n based on relative difference sets in groups of order 8n; this led to the classification of all CHMs of order at most 36. Based on the work of de Launey and Flannery, we describe a classification algorithm for CHMs of order $4p$ with p a prime; we prove refined structure results and provide a classification for p less than or equal to 13. Our analysis shows that every CHM of order $4p$ with p congruent to 1 mod 4 is equivalent to a Hadamard matrix with one of five distinct block structures, including Williamson type and (transposed) Ito matrices. If p is congruent to 3 mod 4, then every CHM of order $4p$ is equivalent to a Williamson type or (transposed) Ito matrix.

Butson Full Propelinear Codes

19 June
16:20

Ronan Egan
Dublin City University, Dublin, Ireland

In this talk I will discuss Butson Hadamard matrices, and codes over finite rings coming from these matrices in logarithmic form, called BH-codes. We introduce a new morphism of Butson Hadamard matrices through a generalized Gray map on the matrices in logarithmic form. That is, we show how, if given a Butson Hadamard matrix over the k^{th} roots of unity, we can construct a larger Butson matrix over the ℓ^{th} roots of unity for any ℓ dividing k, provided that any prime p dividing k also divides ℓ.

We will see that an additive code over the ring of integers modulo q, where q is a power of a prime p, is isomorphic as a group to a BH-code over this ring, and the image of this BH-code under the Gray map is a BH-code over the ring of integers modulo p. Further, we investigate the inherent propelinear structure of these codes (and their images) when the Butson matrix is cocyclic. Some structural properties of these codes are studied and examples are provided.

This is a joint work with José Andrés Armario and Ivan Bailera.

Mutually Orthogonal Binary Frequency Squares

June 29
16:55

Ian Wanless
School of Mathematics, Monash University, Clayton Vic 3800

A *binary frequency square* is a square $(0,1)$-matrix in which each row and column has the same number of zeroes in it. If each row and column has t ones and $n-t$ zeroes then we say the square has frequencies $(n-t,t)$. Two binary frequency squares F_1 and F_2 with frequencies (λ_0,λ_1) and (μ_0,μ_1) are *orthogonal* if for each $\ell,m \in \{0,1\}$ there are $\lambda_\ell\mu_m$ ordered pairs $(F_1[i,j],F_2[i,j]) = (\ell,m)$. A Hadamard matrix of order n can be used for building a "complete" set of $(n-1)^2$ mutually orthogonal binary frequency squares of order n. In this talk I will survey recent work on sets of mutually orthogonal binary frequency squares.

June 30
9:00
online talk

Quasi-Symmetric 2-$(41, 9, 9)$ Designs and Doubly Even Self-Dual Codes of Length 40

Akihiro Munemasa

Graduate School of Information Sciences, Tohoku University, Sendai, Japan

The existence of a quasi-symmetric 2-$(41, 9, 9)$ design with block intersection numbers 1 and 3 is an open problem. Taking the residual gives a quasi-symmetric 1-$(40, 8, 9)$ design with block intersection numbers 0 and 2. The incidence matrix of the latter generates a binary doubly even code of length 40, which is contained in a doubly even self-dual code of length 40. Since binary doubly even self-dual codes of length 40 have been classified by Betsumiya, Harada and the author, it is reasonable to expect that the existence problem of a quasi-symmetric 2-$(41, 9, 9)$ design with block intersection numbers 1 and 3 can be settled. However, because of the large amount of necessary computations, we have been able to prove the non-existence result under the hypotheses that the design in question has a fixed-point-free automorphism of order 5 and 2-rank 20. We will present theoretical results which we used to reduce the amount of necessary computations.

This is based on joint work with Vladimir Tonchev.

June 30
9:50
online talk

Equivalence of the Existence of Hadamard Matrices and Cretan$(4t - 1, 2)$-Mersenne Matrices

Jennifer Seberry

School of Computing and Information Technology, Faculty of Engineering and Information Science, University of Wollongong, Wollongong, NSW 2522, Australia

We study orthogonal matrices whose elements have moduli $\leqslant 1$. This paper shows that the existence of two such families of matrices is equivalent. Specifically we show that the existence of an Hadamard matrix of order $4t$ is equivalent to the existence of a 2-level Cretan-Mersenne matrix of order $4t - 1$.

This work is based on https://arxiv.org/pdf/1501.07012.pdf.

Diagonal Unitary and Orthogonal Symmetries in Quantum Theory

June 30
11:05

Ion Nechita

CNRS Laboratoire de Physique Théorique UMR 5152, 118 route de Narbonne, 31062 Toulouse Cedex, France

We study bipartite operators which stay invariant under the local action of the diagonal unitary and orthogonal groups. We investigate structural properties of these operators, arguing that the diagonal symmetry makes them suitable for analytical study, and that they are a rich source of (couter-)examples in the theory of quantum information. We focus on positive semi-definite and unitary bipartite operators. In particular, for bipartite unitary matrices having the aforementioned symmetry, we show that the maximum entangling power of the gate is achieved when one of the defining matrices is a complex Hadamard matrix. We discuss the corresponding question for real Hadamard matrices, formulating a conjecture.

Classification of Skew-Hadamard Matrices of Small Orders

June 30
11:55
online talk

Behruz Tayfeh-Rezaie

School of Mathematics, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5746, Tehran, Iran

A Hadamard matrix H is said to be skew-Hadamard if $H + H^t = 2I$. Skew-Hadamard matrices, being equivalent to doubly regular tournaments, form a class of Hadamard matrices, which has been widely studied. They are used to construct several combinatorial objects, such as association schemes, self-dual codes, strongly regular graphs, and more. Skew-Hadamard matrices have already been classified up to order 28. Here, we give a complete classification for order 32. As a consequence, the classification of association schemes of order 31 is obtained. In order 36, we provide a complete classification of type one matrices and find many examples of type two.

This is a joint work with A. Hanaki, H. Kharaghani and A. Mohammadian.

June 30
14:45

Hard Combinatorial Problems: 20+ Years of Legendre Pairs

Ilias Kotsireas

Wilfrid Laurier University, Department of Physics and Computer Science, 75 University Avenue West Waterloo, Ontario N2L 3C5, Canada

Legendre pairs were introduced in 2001 by Seberry and her students, as a means to construct Hadamard matrices via a two-circulant core construction. A Legendre pair consists of two sequences of odd length ℓ, with elements from $\{-1, +1\}$, such that their respective autocorrelation coefficients sum to -2, or (equivalently) their respective power spectral density coefficients sum to $2\ell + 2$. Legendre pairs of every odd prime length exist, via a simple construction using the Legendre symbol. We will review known constructions for Legendre pairs. We will discuss various results on Legendre pairs during the past 20 years, including the concept of compression, introduced in a joint paper with Djokovic, as well as the computational state-of-the-art of the search for Legendre pairs. In particular, we recently contributed the only known Legendre pair of length $\ell = 77$ in a joint paper with Turner/Bulutoglu/Geyer. In addition, we recently contributed in a joint paper with Koutschan, several Legendre pairs of new lengths $\ell \equiv 0 \pmod 3$, as well as an algorithm that allows one to determine the full spectrum of values for the $\ell/3$-rd power spectral density value. The importance of Legendre pairs lies in the fact that they constitute a promising avenue to the Hadamard conjecture.

June 30
16:05

Quantum Algorithms and Hadamard Matrices

Māris Ozols

Institute for Logic, Language and Computation (ILLC), Faculty of Science, University of Amsterdam, Science Park 107, 1098 XG Amsterdam

I will discuss a simple but little-known correspondence between Hadamard matrices and exact 1-query quantum algorithms due to Montanaro. It provides a unified framework for a number of known algorithms (e.g. Deutsch, Bernstein-Vazirani, shifted Legendre symbol, etc.) and suggests the possibility of transferring ideas between the two fields. While these algorithms solve seemingly artificial problems, this may change when the problem is viewed from the right angle. For example, the Bernstein-Vazirani algorithm can be reinterpreted as computing the gradient of a function, leading to a quantum speed-up for a fundamental problem. The goal of my talk is to promote this philosophy in the hope of finding new examples of quantum speedups for meaningful problems based on Hadamard matrices.

Balancedly Splittable Orthogonal Designs

June 30
16:40
online talk

Hadi Kharaghani

Department of Mathematics and Computer Science, University of Lethbridge, Lethbridge, Canada

Balancedly splittable orthogonal designs will be introduced togetherwith a recursive construction method. The applications include equiangulartight frames over the real, complex, and quaternions meeting the Delsarte-Goethals-Seidel upper bound.

This is a joint work with Thomas Pender and Sho Suda.

Outstanding Problems in Algebraic Design Theory

July 1
9:00

Dane Flannery

School of Mathematics, Statistics, and Applied Mathematics, National University of Ireland, Galway, Ireland

We discuss several widely varying topics in Algebraic Design Theory, emphasizing foundational questions that still await answers.

2-Unitary Hadamard Matrices and Multipartite Quantum Entanglement

July 1
9:50

Wojciech Bruzda

Institute of Theoretical Physics of Jagiellonian University, ul. prof. S. Łojasiewicza 11, 30-348 Kraków, Poland

We will discuss a special class of (complex) Hadamard matrices and their direct applications in quantum physics. We say that a Hadamard matrix $H = H_{ijkl}$ of square dimension is of 2-unitary type, if it remains Hadamard after two particular rearrangements of 4-index, i.e. H_{ilkj} and H_{ikjl} are (complex) Hadamards too. Such matrices represent Absolutely Maximally Entangled (AME) states in Quantum Information Theory. We will explain the importance of such objects, provide examples, and ask a question about their existence in any dimension.

July 1
10:50
online talk

Partial Difference Sets of Latin Square Type or Negative Latin Square Type in Abelian Non p-Groups

Koji Momihara

Division of Natural Science, Faculty of Advanced Science and Technology, Kumamoto University, 2-40-1 Kurokami, Kumamoto 860-8555, Japan

Davis and Jedwab (1997) established a construction theory unifying previously known constructions of difference sets, relative difference sets and divisible difference sets. They introduced the concept of *building blocks*, which played an important role in a construction of Menon-Hadamard difference sets. On the other hand, Polhill (2010) gave a construction of Paley type partial difference sets (conference graphs) based on a special system of building blocks, called a *covering extended building set*, and proved that there exists a Paley type partial difference set in an abelian group of order $9^i v^4$ for any odd positive integer v and any $i = 0, 1$. His result covers all orders of abelian groups in which Paley type partial difference sets exist. In this talk, we give new constructions of partial difference sets of Latin square type or negative Latin square type in abelian groups by extending the theory of building blocks, which are generalizations of Polhill's result.

July 1
11:40

Grothendieck Inequality: Old and New

Shmuel Friedland

Department of Mathematics, University of Illinois at Chicago,
homepages.math.uic.edu/~friedlan/

In 1953, Grothendieck proved a powerful matrix inequality that he called "the fundamental theorem in the metric theory of tensor products". It states that for $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$, there is a finite constant $K_G > 0$ such that for every $d, m, n \in \mathbb{N}$ with $d \geq m + n$ and every matrix $A = (a_{ij}) \in \mathbb{F}^{m \times n}$,

$$\|A\|_{1,\infty} := \max_{|\varepsilon_i|=|\delta_j|=1} \left| \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \varepsilon_i \bar{\delta}_j \right| \leq \|A\|_G$$

$$:= \max_{\|x_i\|=\|y_j\|=1} \left| \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \langle x_i, y_j \rangle \right| \leq K_G \|A\|_{1,\infty}.$$

where the maximum on the left is taken over all $\delta_i, \varepsilon_j \in \mathbb{F}$ of unit absolute value, $i = 1, \ldots, m$, $j = 1, \ldots, n$, and the maximum in the middle is taken over all $x_i, y_j \in \mathbb{F}^d$ of unit norm. The exact value of K_G, which depends on the choice of $\mathbb{F}$, remains unknown but the best current upper and lower bounds are $1.67696 \leq K_G^{\mathbb{R}} \leq \pi/2 \log(1+\sqrt{2}) \approx 1.78221$ and $1.33807 \leq K_G^{\mathbb{C}} \leq 1.40491$. One main reason behind the power of (1) is that $\|A\|_{1,\infty}$ is NP-hard to compute, while $\|A\|_G$ is computable in polynomial time using semidefinite programming.

I begin by highlighting some important consequences of the Grothendieck inequality in quantum information and combinatorial optimization. I will then describe our recent contributions to various aspects of the Grothendieck inequality including (i) a simple proof [4] of (1) that yields also the current best known upper bounds for K_G by Krivine and Haagerup; (ii) a remarkable discovery [5] that K_G is the norm of Strassen's famous 3-tensor for matrix-matrix product; (iii) an extension of (1) over quaternions [2]; (iv) a generalization of (1) to symmetric matrices [3]; and (v) recent numerical results [1] suggesting that the average value of the Grothendieck constant is close to 1.

The *symmetric Grothendieck inequality* in (iv) deserves special highlight. Let $\mathbb{S}^n$ be the space of $n \times n$ symmetric matrices over $\mathbb{R}$ or Hermitian matrices over $\mathbb{C}$. Then

$$\|B\|_\theta := \max_{|\delta_i|=1} \left| \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} \delta_i \bar{\delta}_j \right| \le \|B\|_\gamma \tag{1}$$

$$:= \max_{\|x_i\|=1} \left| \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} \langle x_i, x_j \rangle \right| \le K_\gamma \|B\|_\theta. \tag{2}$$

We showed in [3]

$$K_\gamma^{\mathbb{R}} \le \sinh \frac{\pi}{2} \approx 2.30130, \qquad K_\gamma^{\mathbb{C}} \le \frac{8}{\pi} - 1 \approx 1.54648.$$

[1] S. Cole and S. Friedland, "Numerical evidence of a concentration law for the Grothendieck constant," *in preparation.*

[2] S. Friedland, Z. Lai, and L.-H. Lim, "Grothendieck's inequality and Haagerup's bound over quaternions," *in preparation.*

[3] S. Friedland and L.-H. Lim, "Symmetric Grothendieck inequality," `arXiv: 2003.07345`, 2020.

[4] S. Friedland, L.-H. Lim, and J. Zhang, "An elementary and unified proof of Grothendieck's inequality," *Enseign. Math.*, **64** (2018), no. 3/4, pp. 327–351.

[5] S. Friedland, L.-H. Lim, and J. Zhang, "Grothendieck constant is norm of Strassen matrix multiplication tensor," *Numer. Math.*, 143 (2019), no. 4, 905-922

Resource Theory of Absolute Negativity

July 1
14:30

Roberto Benjamin Salazar Vargas

Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University, 30-348 Kraków, Poland

A crucial goal of quantum information is to find new ways to exploit the properties of quantum devices as resources. One of the prominent properties of quantum devices of particular interest is their negativity in quasi-probability representations, intensively studied in foundational and practical investigations. A quantum device's most general quasi-probability representation is given by the mathematical formalism of quantum frames, including distributions over mutually unbiased basis (MUBs) and symmetric informationally complete measurements (SIC-POVMs).

In this article, we introduce the concept of Absolute Negativity to characterise the negativity of sets of quantum devices in a basis-independent way. Moreover, we provide a resource theory for our notion of Absolute Negativity, which applies to sets of quantum state-measurement pairs. Additionally, we determine a computationally efficient and complete hierarchy of upper-bounds for resource measures, which allows for estimating the resources of a set of devices. We demonstrate operational interpretations of the resource theory for discrimination and output-estimation advantages. Furthermore, we illustrate the new concepts presented with an exhaustive analysis of a simple case with four qubit state-measurement pairs. Finally, we discuss the potential advantages of applying quantum frames to the construction of resource theories.

This is a joint work with Jakub Czartowski and A. de Oliveira Junior.

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1 July
15:05

Robust Hadamard Matrices and the Unistochasticity Problem

Grzegorz Rajchel-Mieldzioć

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We study a special class of (real or complex) robust Hadamard matrices, distinguished by the property that their projection onto a 2-dimensional subspace forms a Hadamard matrix. It is shown that such a matrix of order n exists, if there exists a skew Hadamard matrix of this size. This is the case for any even dimension $n \leqslant 20$, and for these dimensions we demonstrate that a bistochastic matrix B located at any ray of the Birkhoff polytope, (which joins the center of this body with any permutation matrix), is unistochastic. An explicit form of the corresponding unitarymatrix U, such that $B_{ij} = |U_{ij}|^2$, is determined by a robust Hadamard matrix. These unitary matrices allow us to construct a family of orthogonal bases in the composed Hilbert space of order $n \times n$. Each basis consists of vectors with the same degree of entanglement and the constructed family interpolates between the product basis and the maximally entangled basis.

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1 July
16:05
online talk

Excess of a Matrix: an Application to Quantum Mechanics

Dardo Goyeneche

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In this talk, we extend the notion of excess to a wide class of complex matrices. We show that the problem to determine the maximal excess within an equivalent class of matrices is one-to-one related to the problem of calculating the local-hidden-variable value of Bell inequalities in quantum mechanics. That is, it is equivalent to finding the maximal value that can take a certain function defined by correlations shared by two parties within the framework of a local and deterministic physical theory. Our study also allowed us to find an interesting sequence of tight Bell inequalities based on real Hadamard matrices, thus defining facets of the local-hidden-variable polytope.
