From the Abacus to the Quantum Computer

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- Introduction.
- Calculation of the partition function on a quantum computer and quantum supremacy
- Partition function, Fisher zeros and entanglement
- Conclusions.

Brief History of Computation

- Ancient Tools: Abacus (c. 2300 BCE) and tally systems.
- Mechanical Era: Pascaline (1642), Leibniz calculator (1673), Babbage's Analytical Engine (1830s).
- Electronic Computers: ENIAC (1945), transistor computers (1950s–60s).
- Modern Computing: Microprocessors, PCs, smartphones.
- Quantum Computing: 21st century — a new paradigm using qubits.

Abacus

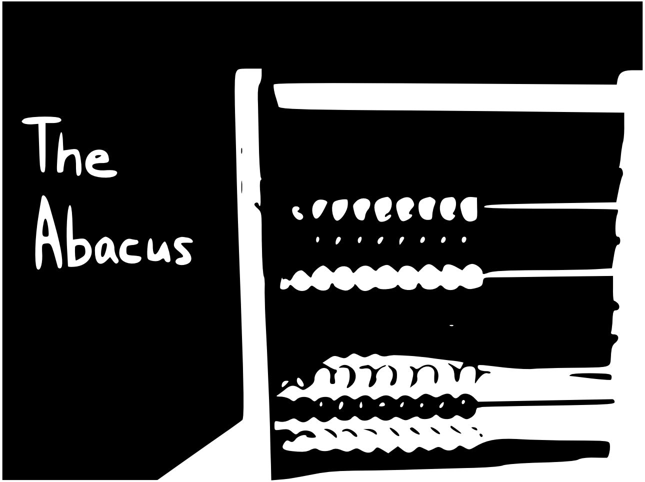


Рис.: One of the first classical computers

Quantum computer

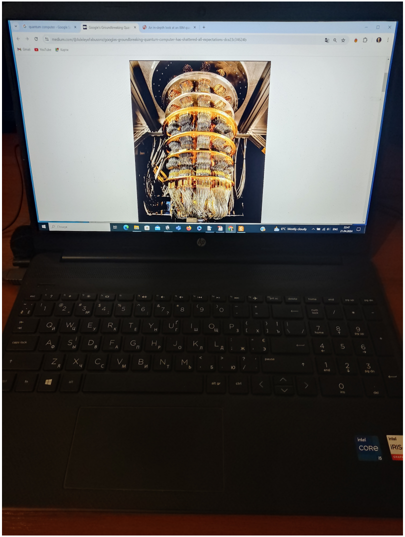


Рис.: Classical and quantum computers

- **A quantum computer** is a device that uses the principles of the quantum world (superposition, entanglement) for computation, a process known as quantum computation.

- Feynman's Paper on Quantum Computers (1982):

Richard P. Feynman, Simulating Physics with Computers, International Journal of Theoretical Physics, Vol. 21, 1982
Section 4 is entitled:
QUANTUM COMPUTERS – UNIVERSAL QUANTUM SIMULATORS

- Famous Quote from the Paper:

Nature isn't classical ... and if you want to make a simulation of nature, you'd better make it quantum mechanical.

From Classical to Quantum bits

Classical bit : *or* 0 *or* 1

$$\begin{pmatrix} \bullet \\ \circ \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle \rightarrow 0 \qquad \begin{pmatrix} \circ \\ \bullet \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle \rightarrow 1$$

Quantum bit (*superposition principle*) : *and* 0 *and* 1

$$|q\rangle = a|0\rangle + b|1\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$

Two classical bits : *or*, *or*, *or*, *or*

$$\begin{pmatrix} \bullet \\ \circ \end{pmatrix}\begin{pmatrix} \bullet \\ \circ \end{pmatrix} \rightarrow |00\rangle, \qquad \begin{pmatrix} \bullet \\ \circ \end{pmatrix}\begin{pmatrix} \circ \\ \bullet \end{pmatrix} \rightarrow |01\rangle,$$

$$\begin{pmatrix} \circ \\ \bullet \end{pmatrix}\begin{pmatrix} \bullet \\ \circ \end{pmatrix} \rightarrow |10\rangle, \qquad \begin{pmatrix} \circ \\ \bullet \end{pmatrix}\begin{pmatrix} \circ \\ \bullet \end{pmatrix} \rightarrow |11\rangle$$

Two quantum bits : *and*, *and*, *and*, *and*
Superposition principle

$$|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle.$$

Entanglement:
State of two qubits can not be represented as states of first qubit and second qubit

$$|\psi\rangle \neq |q_1\rangle|q_2\rangle$$

N classical bits: or, or, or, or,..., or – $\mathbf{2^N}$ states

$$|0000,..,000\rangle, \quad |0000,..,001\rangle, \quad |0100,..,101\rangle, ... \\ |1010,..,010\rangle, \quad |1111,..,111\rangle$$

N quantum bits: and, and, and, and,..., and – $\mathbf{2^N}$ terms
Superposition principle

$$|\psi\rangle = \sum_{x_1=0,1} \sum_{x_2=0,1} ... \sum_{x_N=0,1} c_{x_1,x_2,...,x_N} |x_1, x_2, ..., x_N\rangle =$$
$$a|0000,..,000\rangle + b|0000,..,001\rangle + ... + c|0100,..,101\rangle + ... \\ +d|1010,..,010\rangle + ... + l|1111,..,111\rangle.$$

Simulation of quantum computer with classical one

$$|\psi\rangle = \sum_{x_1,x_2,\ldots,x_N} C_{x_1x_2\ldots x_N}|x_1x_2\ldots x_N\rangle$$

N quantum bits correspond to 2^N classical bits.
IBM quantum computer (Ospray, 2022): number of quantum bits N=433

$$2^N = 2^{433} \simeq 10^{130}$$

In order to simulate Ospray on classical computer we need 10^{130} classical bits

$$10^{130} = 10000000000000000000000000000000000000000\\00000000000000000000000000000000000000000000000000\\0000000000000000000000000000000000000000$$

December 2023, IBM, QC Condor, N=1121 qbits

$$\mathbf{1121\ qbits \rightarrow 2^{1121} \simeq 10^{337}\ classical\ \ bits}$$

Number of nucleons in the Universe is $\mathbf{10^{80}}$.
Even the Universe, as a classical computer, can not simulate QC Condor.

Classical calculation

$$input \;\; |x_1, x_2, ..., x_N\rangle \Rightarrow \begin{pmatrix} Classical \\ computer \end{pmatrix} \Rightarrow output$$

To find min $f(x_1, x_2, ..., x_N)$ it is necessary to perform $\mathbf{2^N}$ calculations.

Quantum calculation

$$input \; |\psi\rangle \Rightarrow \begin{pmatrix} Quantum \\ computer \end{pmatrix} \Rightarrow output|\psi'\rangle \Rightarrow \quad Measurement$$

$$|\psi\rangle = superposition \; of |x_1, x_2, ..., x_N\rangle$$

Quantum parallelism: Quantum computer can perform calculation with different $\mathbf{2^N}$ numbers at the same time for one calculation.

- The art of quantum programming lies in exploiting superposition (quantum parallelism) and entanglement in computation to achieve quantum supremacy.

- Calculation of the partition function is a good example for achieving quantum supremacy.

Calculation of partition function and quantum supremacy

This talk is based on:

• H.P. Laba, V.M. Tkachuk, Calculation of the partition function of the Ising model on a quantum computer, Phys. Lett. A 491 129213 (2023).

• H. P. Laba, V. M. Tkachuk, Quantum computation of Fisher zeros for the Ising model, Phys. Lett. A 534, 130251 (2025).

• A. R. Kuzmak, V. M. Tkachuk, Detecting the purely imaginary Fisher zeros of an Ising spin system on a quantum computer, Sci. Rep. 15, 32854 (2025).

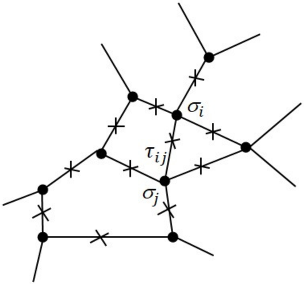
Partition function

$$Z = \mathrm{Tr} e^{-H/T} \tag{1}$$

From Statistical Mechanics: A Set of Lectures
Feynman emphasizes the foundational role of the partition function:
"This fundamental law is the summit of statistical mechanics, and the entire subject is either the slide-down from this summit, as the principle is applied to various cases, or the climb-up to where the fundamental law is derived and the concepts of thermal equilibrium and temperature clarified."

Ising model on a graph

$$H = J \sum_{(i,j)} (\epsilon_{ij} \sigma_i^z \sigma_j^z + 1) \tag{2}$$



Partition function of the Ising model

$$Z = \sum_{\sigma_1,\sigma_2,\ldots\sigma_N} e^{-\beta\sum_{(i,j)}(\epsilon_{ij}\sigma_i\sigma_j+1)}, \tag{3}$$

where $\beta = J/T$ dimensionless inverse temperature, $\sigma_i = \pm 1$.
Classical computation: 2^N terms in sum.

Quantum computation of Tr (Trick 1)

Quantum computation using **quantum parallelism** (superposition) gives result for one calculation

$$Z = \mathrm{Tr} e^{-\beta \sum_{(i,j)} (\epsilon_{ij} \sigma_i^z \sigma_j^z + 1)} = \tag{4}$$

$$= 2^N \langle +, +, ..., + | e^{-\beta \sum_{(i,j)} (\epsilon_{ij} \sigma_i^z \sigma_j^z + 1)} | +, +, ..., + \rangle = \tag{5}$$

$$= 2^N \langle 0, 0, ..., 0 | e^{-\beta \sum_{(i,j)} (\epsilon_{ij} \sigma_i^x \sigma_j^x + 1)} | 0, 0, ..., 0 \rangle, \tag{6}$$

where

$$|+, +, ..., +\rangle = \mathsf{H}_1 \mathsf{H}_2 ... \mathsf{H}_\mathsf{N} |0, 0, ..., 0\rangle = \tag{7}$$

$$\frac{1}{2^{N/2}} \sum_{x_1, x_2, ..., x_N} |x_1, x_2, ..., x_N\rangle \tag{8}$$

and $x_i = 0, 1$.

The main problem is to simulate on a quantum computer the Boltzmann factor $e^{-\beta H}$ that is not a unitary operator. We propose method which are based on using ancilla qubits that allows

$$e^{-\beta H} \longleftrightarrow e^{-itH_{eff}}$$

Realization of the Boltzmann factor on a quantum computer (Trick 2)

Let us consider the Boltzmann factor corresponding to two interacting spins. One can verify by acting on basis vectors the following identity

$$e^{-\beta(\epsilon_{ij}\sigma_i^z\sigma_j^z+1)} = \cos(t(\epsilon_{ij}\sigma_i^z\sigma_j^z+1)) = \frac{1}{2}\mathrm{Tr}_{\tau_{ij}} e^{-it\tau_{ij}^z(\epsilon_{ij}\sigma_i^z\sigma_j^z+1)}, \quad (9)$$

where $\epsilon_{ij} = \pm 1$ and β and t are related by equation

$$e^{-2\beta} = \cos(2t). \quad (10)$$

Note that $\beta = 0$ corresponds $t = 0$ and $\beta \to \infty$ corresponds $t \to \pi/4$. Here we introduce ancilla qubit τ_{ij}^z, which is related to the link between i and j spins.

The Boltzmann factor for the Ising model can be represented as a trace over ancilla spins of a unitary operator

$$e^{-\beta \sum_{(i,j)} (\epsilon_{ij} \sigma_i^z \sigma_j^z + 1)} = \frac{1}{2^{N_e}} \mathrm{Tr}_\tau e^{-it \sum_{(i,j)} \tau_{ij}^z (\epsilon_{ij} \sigma_i^z \sigma_j^z + 1)},$$

where N_e is the number of edges in the graph that is equal to the number of ancilla spins.

Trick 1+ Trick 2

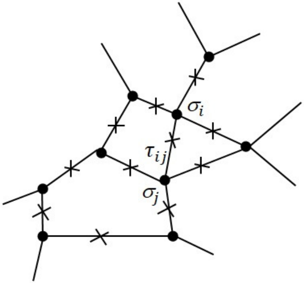
The partition function can be represented as the mean value of the evolution operator

$$Z = 2^N \langle \mathbf{0}, \mathbf{0} | e^{-i\tilde{H}_{\text{eff}}} | \mathbf{0}, \mathbf{0} \rangle, \tag{11}$$

were new effective Hamiltonian reads

$$\tilde{H}_{\text{eff}} = \gamma \sum_{(i,j)} (\epsilon_{ij}\sigma_i^x\sigma_j^x + 1) + t \sum_{(i,j)} \tau_{ij}^x(\epsilon_{ij}\sigma_i^x\sigma_j^x + 1), \tag{12}$$

where we use the fact that $\mathsf{H}\sigma^z\mathsf{H} = \sigma^x$.



Quantum computing of squared modulus of Z

On the quantum computer, we find

$$P_{\mathbf{0},\mathbf{0}} = |\langle \mathbf{0}, \mathbf{0}| e^{-i\tilde{H}_{\text{eff}}} |\mathbf{0}, \mathbf{0}\rangle|^2, \tag{13}$$

which is the probability of obtaining the state $|\mathbf{0}, \mathbf{0}\rangle$ after the measurement during the evolution with the effective Hamiltonian.

For squared modulus of partition function we have

$$|Z|^2 = 2^{2N} P_{\mathbf{0},\mathbf{0}} = 2^{2N} \frac{N_{\mathbf{0},\mathbf{0}}}{N_m}, \tag{14}$$

where N_m is the number of measurements (shots) performed on a quantum computer, and $N_{\mathbf{0},\mathbf{0}}$ is the number of results with the state $|\mathbf{0}, \mathbf{0}\rangle$ obtained during these measurements.

Two-spin Ising cluster. Fisher zeros.

$$H = J(\sigma_1^z \sigma_2^z + 1), \tag{15}$$

where $\epsilon_{12} = 1$ and $J > 0$. Note that, according to (??), for the positivity of H we have added the constant J. The squared modulus of the partition function for this model with a complex inverse temperature $\frac{J}{T} = \beta + i\gamma$ reads

$$|Z|^2 = 2^4 e^{-2\beta} (\sinh^2 \beta + \cos^2 \gamma). \tag{16}$$

The real part of the inverse temperature β and the evolution parameter t are related according to equation (10).
Solutions of $|Z|^2 = 0$ give Fisher zeros. For the two-spin cluster, the Fisher zeros are at the real part of the inverse temperature $\beta = 0$ and the imaginary one $\gamma = \pi(n + 1/2)$.

For the calculation of the squared modulus of the partition function on the quantum computer according to (28), we use

$$|Z|^2 = 2^4 P_{\mathbf{0,0}} = 2^4 \frac{N_{\mathbf{0,0}}}{N_m} = \frac{N_{\mathbf{0,0}}}{2^6}, \tag{17}$$

where the number of spins is $N = 2$ and the number of measurements for each temperature is $N_m = 1024$, $P_{\mathbf{0,0}}$ is given by (27) with the effective Hamiltonian (12) which now is

$$\tilde{H}_{\text{eff}} = t\sigma_0^x(\sigma_1^x\sigma_2^x + 1) + \gamma(\sigma_1^x\sigma_2^x + 1), \tag{18}$$

where, for simplification of notation, we denote the ancilla spin as follows, $\tau_{12}^x = \sigma_0^x$.

The quantum algorithm for calculating the partition function for two-spin cluster

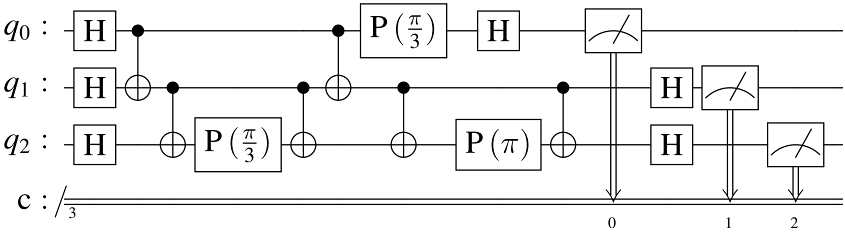


Рис.: Quantum circuit for the calculation of the squared modulus of the partition function of a two-spin cluster, where q_0 is the ancilla qubit corresponding to the ancilla spin σ_0, qubit q_1 corresponds to the spin σ_1 and qubit q_2 corresponds to the spin σ_2. Here, the parameters are $2t = \pi/3$ and $2\gamma = \pi$.

The result of the quantum computing for the two-spin cluster on the 127-qubit IBM-Sherbrooke quantum computer is presented in Fig. 2.

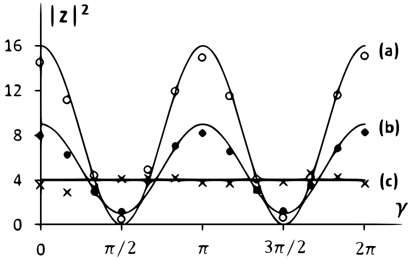


Рис.: Squared modulus of the partition function of the two-spin cluster for different real parts of the inverse temperature β as a function of the imaginary part of the inverse temperature γ. The lines mark the analytical results. The results of the quantum calculation on the quantum computer are marked by $\circ$, $\bullet$, and $\times$. Line (a) and $\circ$ correspond to $\beta = 0$ $(t = 0)$, line (b) and $\bullet$ correspond to $e^{-2\beta} = 1/2$ $(t = \pi/6)$, line (c) and $\times$ correspond to $\beta = \infty$ $(t = \pi/4)$.

Imaginary-temperature partition function

Consider the spin 1/2 system described by the Ising-type Hamiltonian

$$\hat{H}_z = H(\sigma_1^z, \sigma_2^z, ..., \sigma_N^z), \tag{19}$$

where H is some function of the z component of spin operators.

The partition function with imaginary temperature $\beta = it$ reads

$$Z = \mathrm{Tr}_\sigma e^{-it\hat{H}_z} = \sum_{\mathbf{x}} \langle \mathbf{x} | e^{-it\hat{H}_z} | \mathbf{x} \rangle, \tag{20}$$

where t is the actual time of evolution. Here we introduce notation $|\mathbf{x}\rangle = |x_1, x_2 ... x_N\rangle$ with $x_j = 0, 1$ (that corresponds to spin notation $\uparrow, \downarrow$) are the eigenstates of the Hamiltonian with eigenvalues $H = H(\sigma_1, \sigma_2, ..., \sigma_N)$, $\sigma_j^z |x_j\rangle = \sigma_j |x_j\rangle$ with $\sigma_j = 1$ for $x_j = 0$ and $\sigma_j = -1$ for $x_j = 1$.

Tracing over the eigenstates of the Hamiltonian gives the classical expression for the partition function with complex temperature

$$Z = \sum_{\sigma_1=\pm 1} \sum_{\sigma_2=\pm 1} \dots \sum_{\sigma_N=\pm 1} e^{-itH}, \tag{21}$$

where Hamiltonian $H = H(\sigma_1, \sigma_2, ..., \sigma_N)$ is a function of discrete variables $\sigma_1, \sigma_2, ..., \sigma_N$ each of wich take to values ± 1. Note that this summation contains 2^N terms.
Therefore, the calculation of this sum for large N with the help of a classical computer is not a simple task.
In this paper, we show that a quantum computer using quantum parallelism allows us to calculate the partition function of this type for one calculation.

Partition function and Fidelity

Using the orthogonality of quantum states $\langle \mathbf{x}'|\mathbf{x}\rangle = \delta_{\mathbf{x}'\mathbf{x}}$ the partition function can be written as follows

$$Z = \sum_{\mathbf{x}'} \sum_{\mathbf{x}} \langle \mathbf{x}'| e^{-it\hat{H}_z} |\mathbf{x}\rangle.$$

Let us take into account that

$$\sum_{\mathbf{x}} |\mathbf{x}\rangle = 2^{\frac{N}{2}} \mathsf{H}^{[N]} |\mathbf{0}\rangle, \tag{22}$$

where $\mathsf{H}^{[N]} = \mathsf{H}_1 \mathsf{H}_2 ... \mathsf{H}_N$ are Hadamard operators. Then, the partition function can be written in the form

$$Z = 2^N \langle \mathbf{0}| e^{-it\hat{H}_x} |\mathbf{0}\rangle. \tag{23}$$

Here, we introduce the new Hamiltonian with the $x - x$ interaction

$$\hat{H}_x = \mathsf{H}^{[N]} \hat{H}_z \mathsf{H}^{[N]} = H(\sigma_1^x, \sigma_2^x, ..., \sigma_N^x), \tag{24}$$

where we use the fact that $\mathsf{H}\sigma^z\mathsf{H} = \sigma^x$.

Expression (23) can be written in the form

$$Z = 2^N \langle \mathbf{0} | \psi(t) \rangle, \tag{25}$$

where

$$|\psi(t)\rangle = e^{-it\tilde{H}} |\mathbf{0}\rangle \tag{26}$$

is the evolution graph state.

The quantum computer enables us to determine the fidelity of a quantum state during evolution, which is equal to the probability $P_{\mathbf{0}}$ of obtaining the state $|\mathbf{0}\rangle$ following a measurement during its evolution

$$F(t) = |\langle \mathbf{0} | \psi(t) \rangle|^2 = |\langle \mathbf{0} | e^{-it\tilde{H}} | \mathbf{0} \rangle|^2 = P_{\mathbf{0}}. \tag{27}$$

Taking into account (23) we find

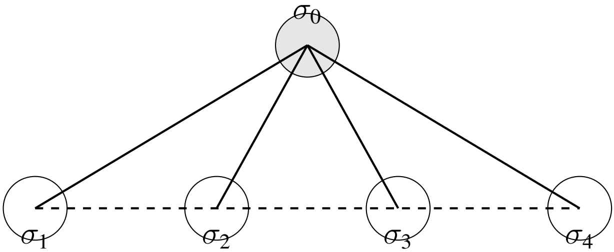
$$|Z|^2 = 2^{2N} F(t) = 2^{2N} \frac{N_{\mathbf{0}}}{N_m}, \tag{28}$$

where $N_{\mathbf{0}}$ represents the number of occurrences of the state $|\mathbf{0}\rangle$ observed out of the total number of measurements N_m performed on the quantum computer.

Ancilla qubit, Partition function, Entanglement

The partition function can be expressed as the mean value of an ancilla spin interacting with the system under consideration, whose total Hamiltonian is given as follows

$$\hat{H}_T = \sigma_0^x \hat{H}_x = \sigma_0^x H(\sigma_1^x, \sigma_2^x, ..., \sigma_N^x). \tag{29}$$



Graph of the spin system: ancilla spin σ_0 interacts with all system spins σ_i, and the spins also interact with each other.

Let us consider the evolution of a quantum state governed by this Hamiltonian starting from the initial state $|0\rangle|\mathbf{0}\rangle$, where $|0\rangle$ is the ancilla state and $|\mathbf{0}\rangle$ is the state of N spins. At the time $t/2$, the quantum state reads

$$|\psi(t/2)\rangle = e^{-i(t/2)\hat{H}_T}|0\rangle|\mathbf{0}\rangle. \tag{30}$$

For the mean value of the z component of the ancilla spin in state $|\psi(t/2)\rangle$, we find

$$\langle\sigma_0^z\rangle = \langle\mathbf{0}|\langle 0|e^{i(t/2)\hat{H}_T}\sigma_0^z e^{-i(t/2)\hat{H}_T}|0\rangle|\mathbf{0}\rangle. \tag{31}$$

Note that σ_0^z anticommutes with $\hat{H}_T$, $\{\sigma_0^z, \hat{H}_T\} = 0$. Therefore $e^{i(t/2)\hat{H}_T}\sigma_0^z = \sigma_0^z e^{-i(t/2)\hat{H}_T}$ and mean value reads

$$\langle\sigma_0^z\rangle = \langle\mathbf{0}|\langle 0|\sigma_0^z e^{-it\sigma_0^x\hat{H}_x}|0\rangle|\mathbf{0}\rangle = \langle\mathbf{0}|\langle 0|e^{-it\sigma_0^x\hat{H}_x}|0\rangle|\mathbf{0}\rangle, \tag{32}$$

where $\langle 0|\sigma_0^z = \langle 0|$.

To provide the next consideration, we represent the ancilla state over the eigenstates of σ_0^x, namely, $\sigma_x|+\rangle = |+\rangle$, $\sigma_x|-\rangle = -|-\rangle$,

$$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle), \tag{33}$$

$$|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle). \tag{34}$$

Then, substituting ancilla state $|0\rangle$ into (32) we have

$$\langle\sigma_0^z\rangle = \frac{1}{2}\left(\langle\mathbf{0}|e^{-it\hat{H}_x}|\mathbf{0}\rangle + \langle\mathbf{0}|e^{it\hat{H}_x}|\mathbf{0}\rangle\right) = \tag{35}$$

$$= \frac{1}{2}\left(\frac{1}{2^N}Z + \frac{1}{2^N}Z^*\right) = \frac{1}{2^N}\mathrm{ReZ}. \tag{36}$$

Similarly, let us consider the mean value of the y component of the ancilla spin

$$\langle \sigma_0^y \rangle = \langle \mathbf{0} | \langle 0 | \sigma_0^z e^{-it\sigma_0^x \hat{H}_x} | 0 \rangle | \mathbf{0} \rangle = -i \langle \mathbf{0} | \langle 1 | e^{-it\sigma_0^x \hat{H}_x} | 0 \rangle | \mathbf{0} \rangle = \tag{37}$$

$$\frac{-i}{2} \left(\langle \mathbf{0} | e^{-it\hat{H}_x} | \mathbf{0} \rangle - \langle \mathbf{0} | e^{it\hat{H}_x} | \mathbf{0} \rangle \right) = \tag{38}$$

$$= \frac{1}{2i} \left(\frac{1}{2^N} Z - \frac{1}{2^N} Z^* \right) = \frac{1}{2^N} \mathrm{Im} Z, \tag{39}$$

where we take into account that $\{\sigma_0^y, \hat{H}_T\} = 0$ and $\langle 0 | \sigma_0^y = -i \langle 1 |$.
Note that σ_0^x commutes with $\hat{H}_T$. As a result, we find $\langle \sigma_0^x \rangle = 0$.

Partition function and mean value of ancilla spin

Thus, we find that the partition function is determined by the mean value of ancilla spins

$$\frac{1}{2^N} Z = \langle \sigma_0^z \rangle + i \langle \sigma_0^y \rangle \tag{40}$$

and

$$\frac{1}{2^{2N}} |Z|^2 = \langle \sigma_0^z \rangle^2 + \langle \sigma_0^y \rangle^2 = \langle \boldsymbol{\sigma} \rangle^2. \tag{41}$$

These formulas relate the partition function to the mean value of the ancilla spin.

Partition function and entanglement

The entanglement of one spin with others in the system is totally determined by its mean value. For instance, the geometric measure of entanglement reads

$$E = \frac{1}{2}(1 - |\langle \boldsymbol{\sigma} \rangle|) \tag{42}$$

and thus, the entanglement and the partition function are related as follows

$$E = \frac{1}{2}\left(1 - \frac{1}{2^N}|Z|\right). \tag{43}$$

Zeros of the partition function correspond to maximal entanglement.

Example of pureli imaginary Fisher Zeros

The triangle spin cluster

$$H = \frac{J}{4}\left(\sigma_1^z\sigma_2^z + \sigma_2^z\sigma_3^z + \sigma_1^z\sigma_3^z\right) + \frac{h}{2}\left(\sigma_1^z + \sigma_2^z + \sigma_3^z\right).$$

The partition function for imaginary temperature $\beta \to i\beta$.

$$\frac{Z(\beta)}{2^4} = \frac{1}{4}\left(e^{-i3J\beta/4}\cos\left(3h\beta/2\right) + 3e^{iJ\beta/4}\cos\left(h\beta/2\right)\right).$$

Zeros are $h\beta = (2n+1)\pi$.

We compute $|Z(\beta)|^2$ as a function of $J\beta$ on the ibm-lagos quantum computer in the case of $h = J$.

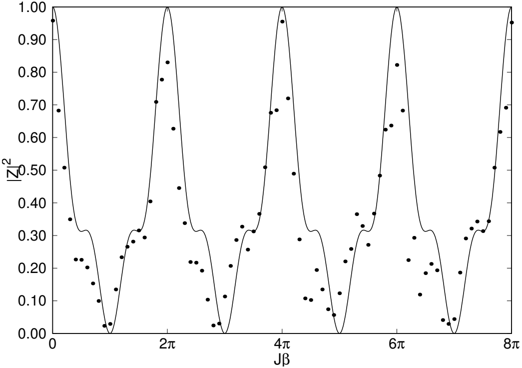


Рис.: The square of the modulus of the partition function is plotted as a function of $J\beta$ in the case of $h = J$. The solid lines represent the theoretical predictions, while the dots depict the results obtained on the ibm-lagos quantum computer. The purely imaginary Fisher zeros correspond to the points at $J\beta = \pi,\ 3\pi,\ 5\pi,$ and 7π.

Possibility to achieve quantum advantagee

The partition function of the Ising model contains 2^N terms $= 2^N$ classical calculations.

A single quantum calculation using superposition yields the partition function.
For $N = 1000$ spins, total number of terms in the partition function is

$$2^{1000} \sim 10^{301}$$

• Approximately 10^{24} bits are currently stored in all digital devices on the planet.
• Total nucleons on Earth $\sim 10^{51}$
• Number of nucleons in Universe $\sim 10^{80}$

Conclusions

- The key quantum resources for achieving quantum supremacy (quantum advantage) are superposition, entanglement, and quantum measurement.
- The quantum computation of the partition function provides a potential route to achieving quantum supremacy.
- Imaginary-temperature partition function $\rightarrow$ mean value of ancilla spin $\rightarrow$ entanglement of the ancilla spin with the system.
- Fisher zeros $\rightarrow$ maximally entangled states.

Thank you for your attention!