Weekly SEMINAR: Quantum Chaos and Quantum Information

Bell correlations and quantum chaos

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Uniwersytet Jagielloński, Kraków

6th October 2025

Outline

1. Bell correlations
2. Quantum chaos
3. Bell correlations and quantum chaos

1. Bell correlations

Example

Classical v/s quantum correlations

A. Kumar Srivastava, GMR, M. Lewenstein et G. Rajchel-Mieldzioć, arXiv:2402.09523 (2024)

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|+1\rangle \otimes |-1\rangle - |-1\rangle \otimes |+1\rangle)$$

$$\hat{\rho} = \frac{1}{2}(|+1\rangle\langle+1| \otimes |-1\rangle\langle-1| + |-1\rangle\langle-1| \otimes |+1\rangle\langle+1|) \ ,$$

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H => A prepares +1 & B prepares -1
T => A prepares -1 & B prepares +1

explained by shared randomness + “list of instructions“

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(+ 1Z, + 1X) do not exist, Z and X are incompatible

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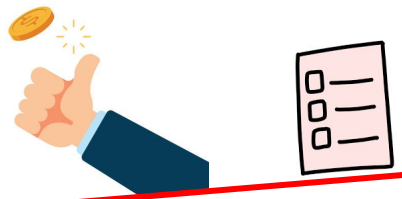

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…

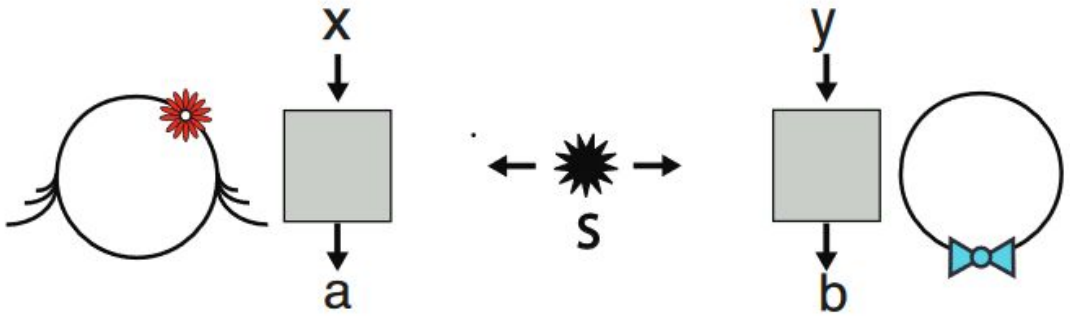
~~explained by shared randomness + “list of instructions“~~

The outcome of a measurement do not “preexist”

Geometry of correlations

Crash course on Bell nonlocality – Bipartite scenario

Bell scenario (device-independent)



After several rounds of collecting statistics, they are able to infer the **joint probability distribution**

$$p(ab|xy)$$

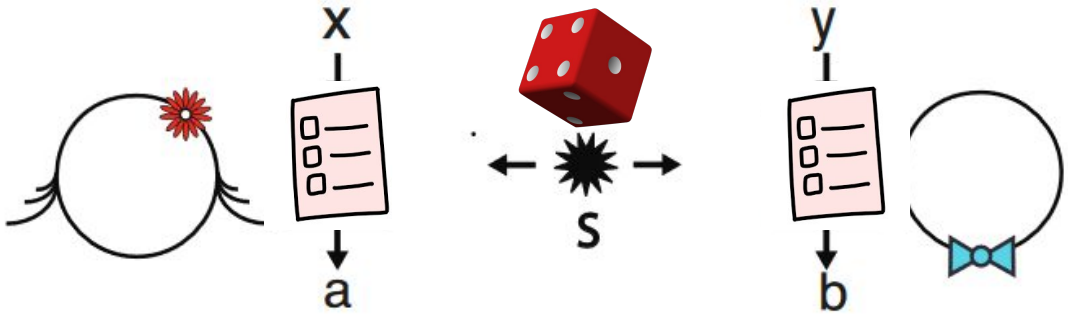
and notice some correlations: p(ab|xy)!=p(a|x)p(b|y)

What can we say about S?

N. Brunner, D. Cavalcanti, S Pironio et al., Rev. Mod. Phys. **86**, 419 (2014)

Crash course on Bell nonlocality

Correlations assisted by shared randomness



Local hidden variable models

$$p(ab|xy) = \int q(\lambda)p(a|x,\lambda)p(b|y,\lambda)d\lambda$$

Crash course on Bell nonlocality

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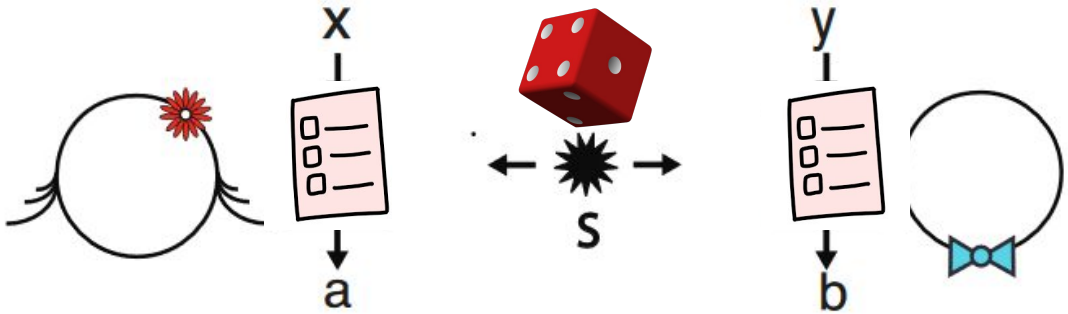
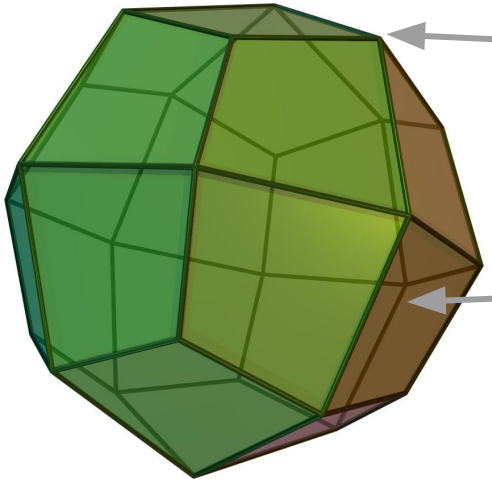




Vertices: Local deterministic strategies

$$p(ab|xy) = \delta_{a|x}\delta_{b|y}$$

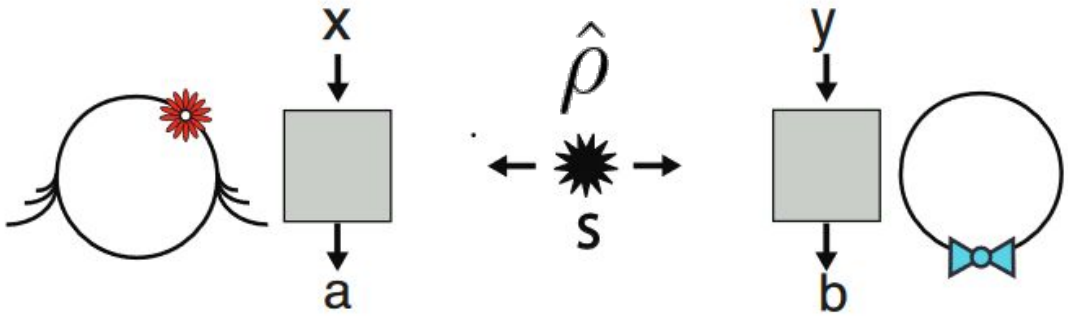
Facets: Bell inequalities

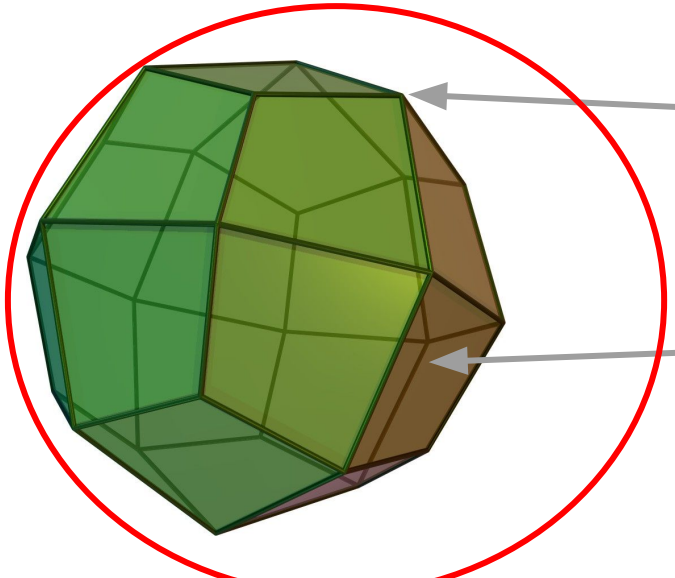
$$\sum_{a,b,x,y} c_{abxy}p(ab|xy) \geq \beta$$

Crash course on Bell nonlocality

Quantum correlations

$$p(ab|xy) = \mathrm{Tr}(\hat{\pi}_{a|x}\hat{\pi}_{b|y}\hat{\rho})$$





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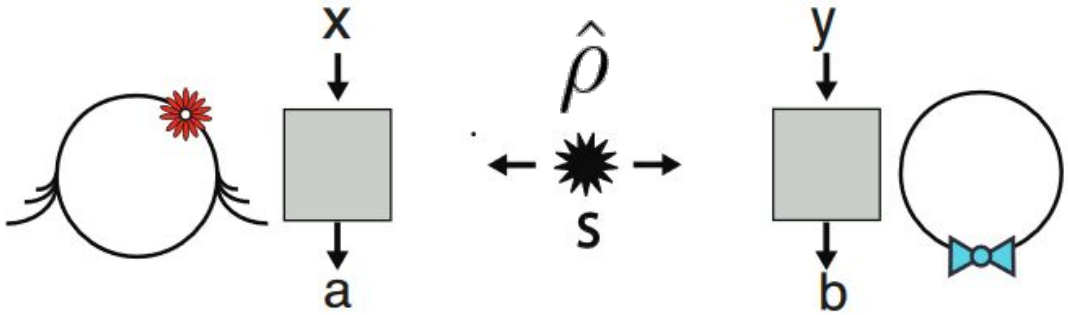
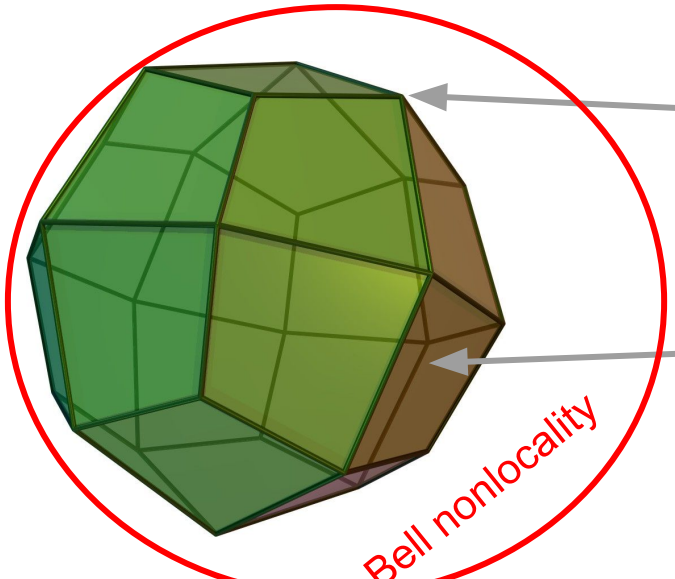




Vertices: Local deterministic strategies

$$p(ab|xy) = \delta_{a|x}\delta_{b|y}$$ Insufficient

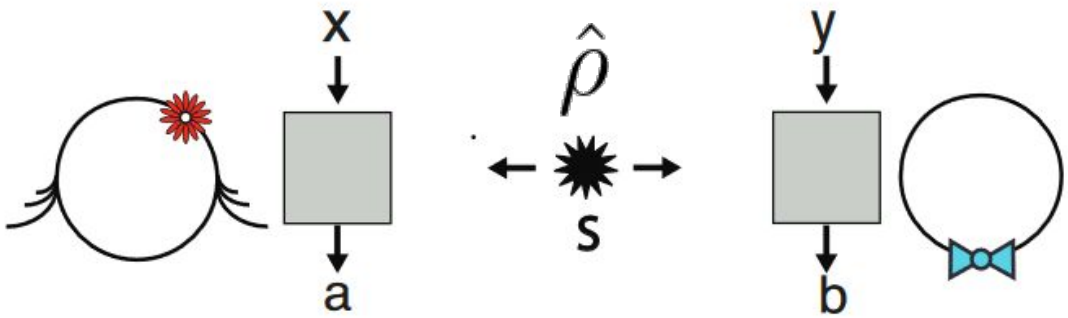
Facets: Bell inequalities

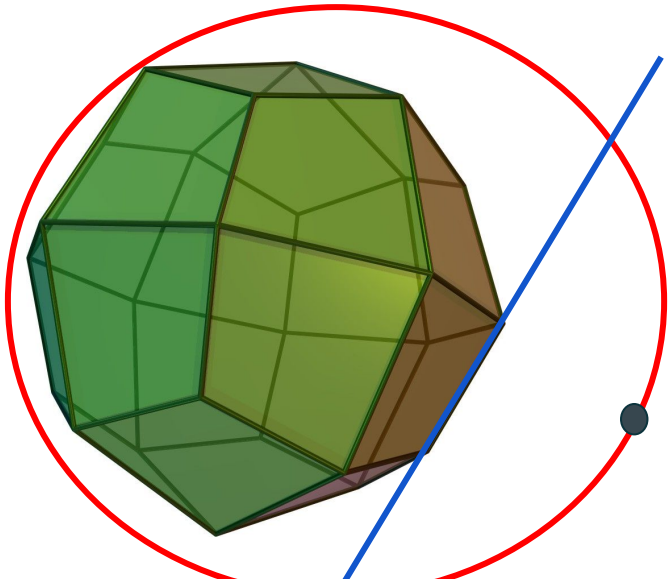
$$\sum_{a,b,x,y} c_{abxy}p(ab|xy) \not\geq \beta$$ Violation

Crash course on Bell nonlocality

Quantum correlations

$$p(ab|xy) = \mathrm{Tr}(\hat{\pi}_{a|x}\hat{\pi}_{b|y}\hat{\rho})$$





CHSH

$$\langle a_x b_y \rangle = \sum_{a,b} ab\, p(ab|xy) \qquad \text{(a,b = +1, -1)}$$

$$\langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle \leq 2$$

Maximally violated by the singlet $|\Psi^-\rangle$ for certain optimized measurement settings

Bell nonlocality => Entanglement

Towards multipartite/many-body systems

REPORT

Detecting nonlocality in many-body quantum states

J. TURA, R. AUGUSIAK, A. B. SAINZ, T. VÉRTESI, M. LEWENSTEIN, AND A. ACÍN Authors Info & Affiliations

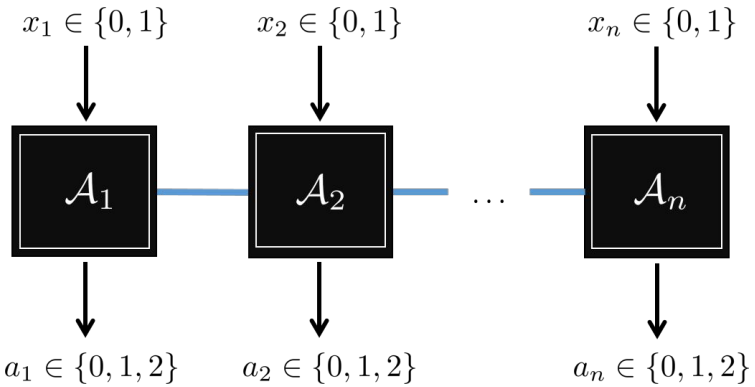
SCIENCE · 13 Jun 2014 · Vol 344, Issue 6189 · pp. 1256-1258 · DOI: 10.1126/science.1247715

Testing nonlocality for many particles

Distant parts of a quantum-mechanical system can be correlated in ways that cannot be described classically—a concept known as nonlocality. Tura *et al.* propose a simple test for nonlocality in systems with multiple particles. The test involves quantities that should readily be measurable in, for example, cold atom experiments. This is an improvement over currently available tests, which are difficult to implement experimentally.

Science, this issue p. 1256

Multipartite scenario

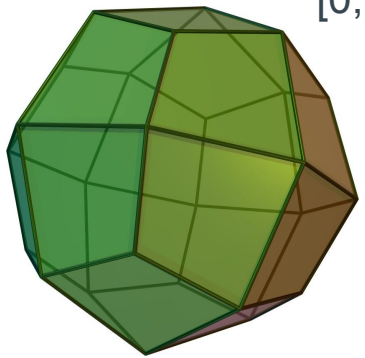


(n, 2, 3)

Infer two body correlations

$$\{p_{ij}(ab|xy)\}_{i\neq j\in[n]}$$

[0,0,1,0,..]



dim ~ n^2

Multipartite scenario – Symmetrization

Permutation-invariant Bell inequalities (PIBIs)

Permutation-invariant (PI)

$$\mathcal{P}_{ab|xy} = \sum_{i \neq j \in [n]} p_{ij}(ab|xy)$$

dim ~ 1

[c1,c2,..]

Infer two body correlations

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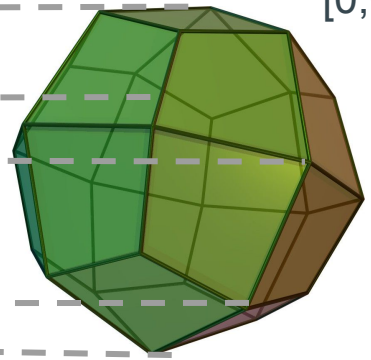

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Multipartite scenario – Bell correlation

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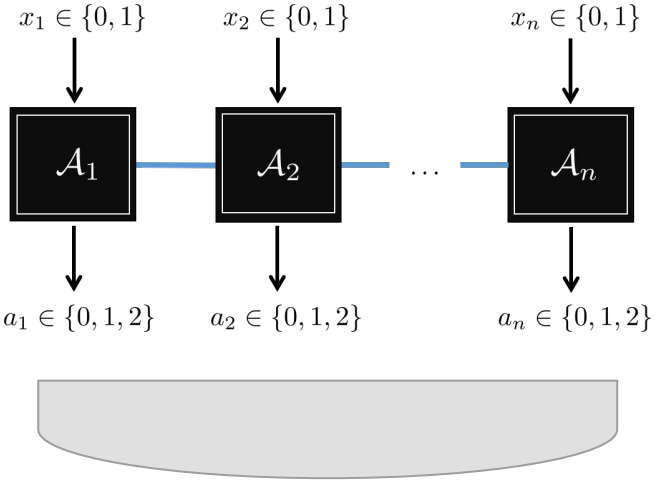
Local measurements (not scalable)



Collective measurements (scalable)

Multipartite scenario – Bell correlation

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Bell correlation instead of **Bell nonlocality**

EDITORS' SUGGESTION

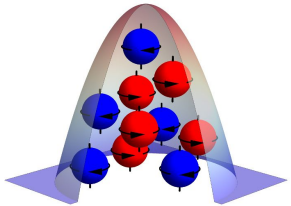
Bell Correlations in Spin-Squeezed States of 500 000 Atoms

Nils J. Engelsen, Rajiv Krishnakumar, Onur Hosten, and Mark A. Kasevich*

Show more

Phys. Rev. Lett. **118**, 140401 – **Published 3 April, 2017**

DOI: https://doi.org/10.1103/PhysRevLett.118.140401



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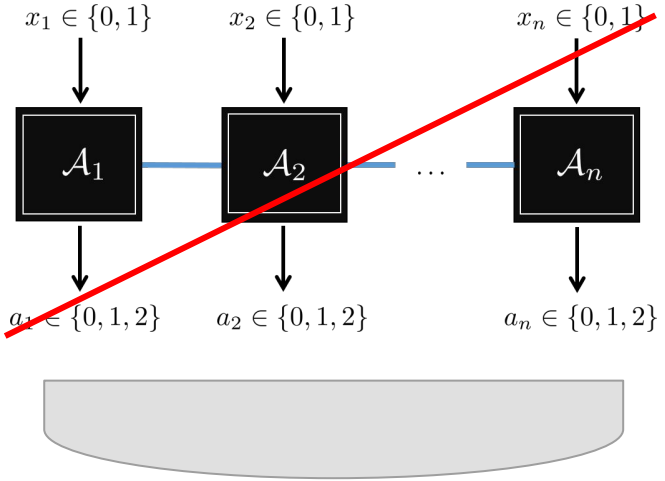

Collective measurements (scalable)

2. Quantum chaos

Classical chaos

Definition: A classical dynamic system is chaotic when small perturbations on initial conditions may have dramatic effects in the long run (Butterfly effect)

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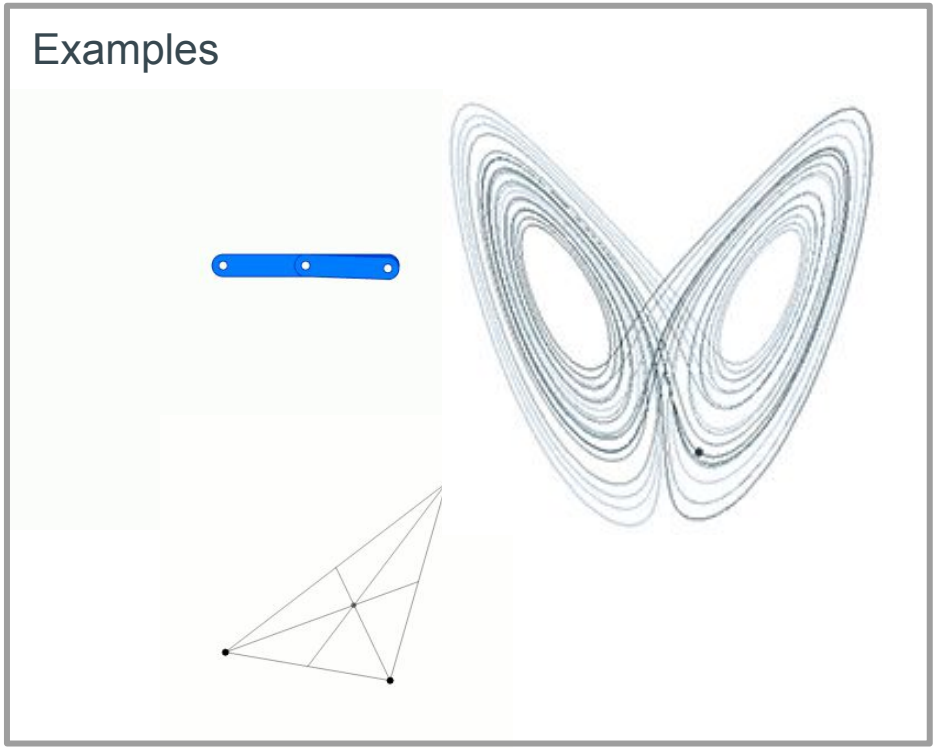
Examples



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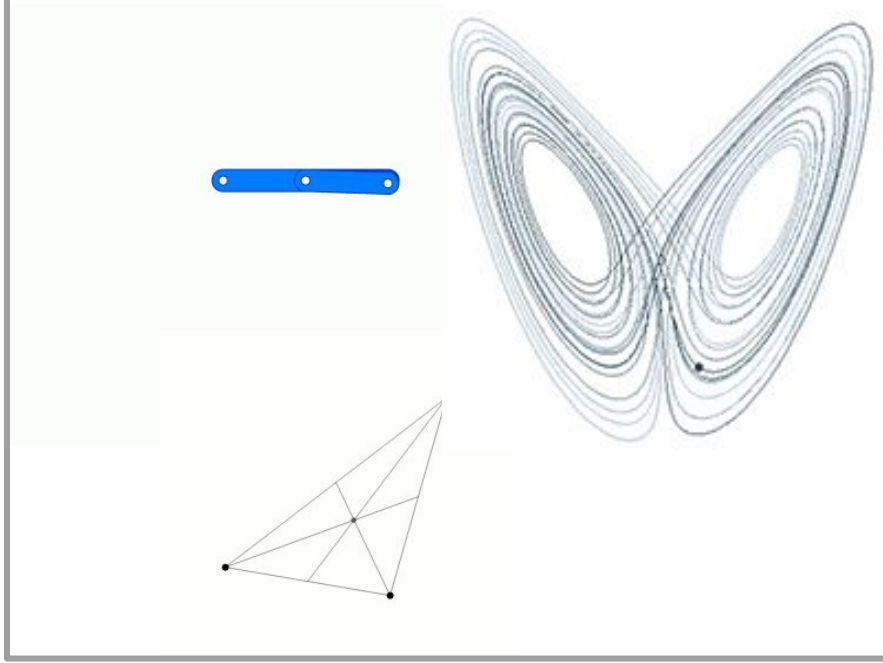
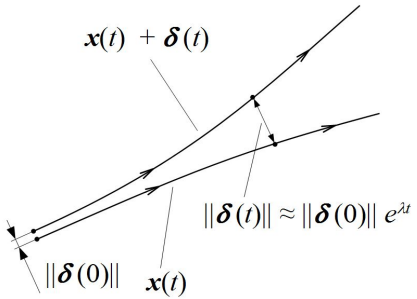


Quantified by Lyapunov exponent

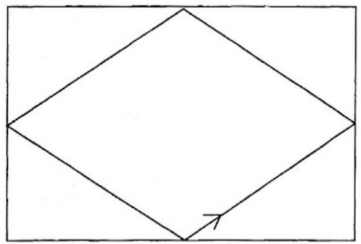


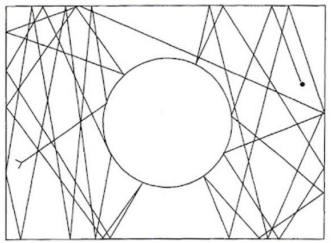
sensitivity on initial conditions

Features of classical chaotic systems

Ergodicity
With enough time, the system will visit all accessible points in the phase space in a uniform way.
Time average = ensemble average
Randomness

Example: free particle/Sinai billiards





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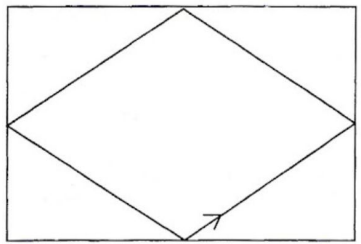
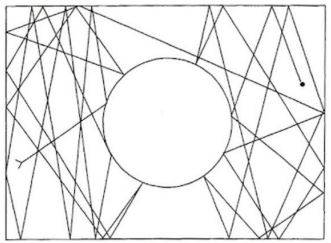
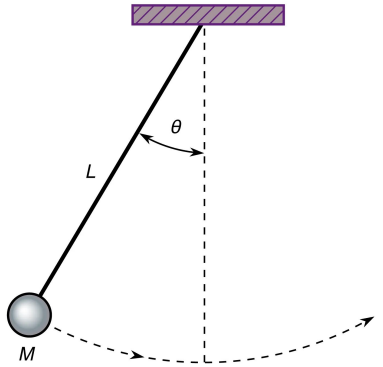




Non-integrability
DOFs > # conserved quantities
Integrability:
DOFs = # conserved quantities
Example: 1-DOF closed systems



Quantum chaos

Problem: phase space trajectories are not well-defined in QM

Quantum chaos

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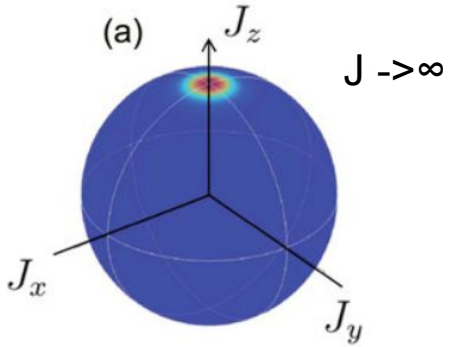
Quantum system is chaotic when it its classical limit is chaotic.

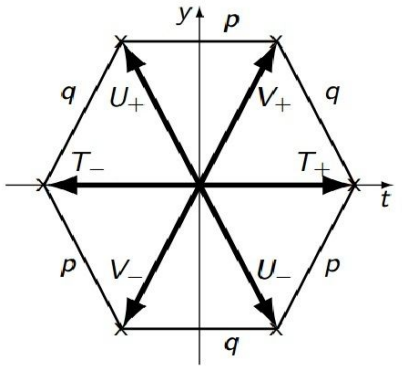
two-level systems SU(2)



J ->∞

1-DOF (1 canonical pair) => integrable

three-level systems SU(3)



p,q ->∞

various DOFs depending on how the classical limit is approached => chaos?

Tools to diagnose quantum chaos in the quantum regime?

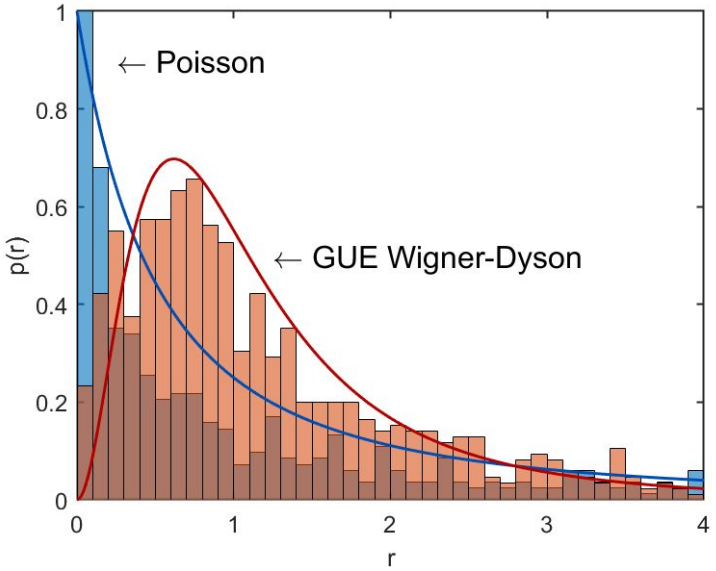
Energy levels spacing distributions

Bohigas-Giannoni-Schmidt conjecture:
Quantum systems with classically chaotic counterparts exhibit energy spectra resembling those of random matrix theory

Energy levels spacing distributions

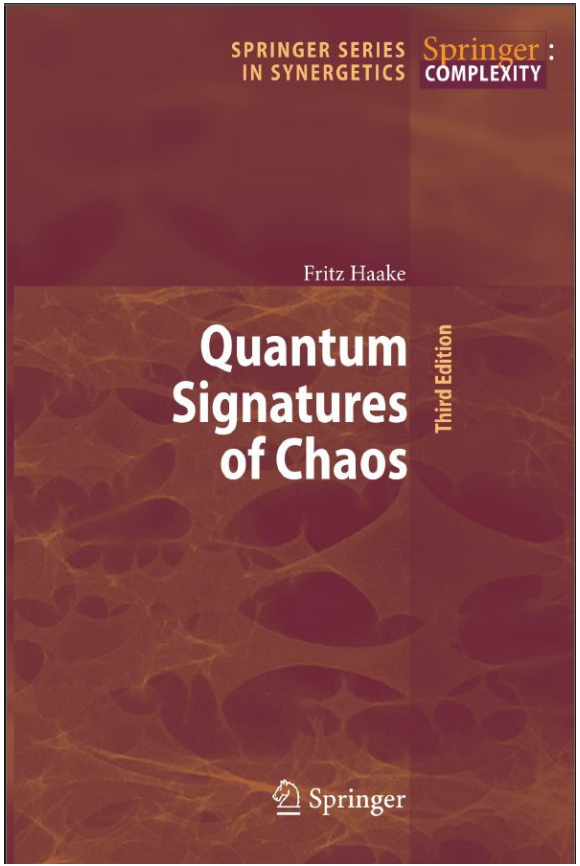
Bohigas-Giannoni-Schmidt conjecture:
Quantum systems with classically chaotic counterparts exhibit energy spectra resembling those of random matrix theory

Example: Ratio of Consecutive level Spacing (RCS) distributions



Poisson
Signature of integrability in the classical limit

Wigner:
Signature of non-integrability/chaos, as described by random matrix theory



IOPscience

Journal of Physics A: Mathematical and General

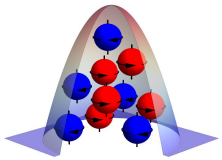
Quantum chaos of SU_3 observables

Sven Gnutzmann, Fritz Haake and Marek Kus

Journal of Physics A: Mathematical and General, Volume 33, Number 1

Citation Sven Gnutzmann *et al* 2000 *J. Phys. A: Math. Gen.* **33** 143

DOI 10.1088/0305-4470/33/1/309



PRL **111,** 090404 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Quantum Chaos in SU(3) Models with Trapped Ions

Tobias Graß,[1] Bruno Juliá-Díaz,[1,2] Marek Kuś,[3] and Maciej Lewenstein[1,4]

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A scheme to generate long-range spin-spin interactions between three-level ions in a chain is presented, providing a feasible experimental route to the rich physics of well-known SU(3) models. In particular, we demonstrate different signatures of quantum chaos which can be controlled and observed in experiments with trapped ions.

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3. Bell correlations and quantum chaos

Integrability and Quantum Chaos in the Spectrum of Bell Operators

Albert Aloy,[1,2,*] Guillem Müller-Rigat,[3] Maciej Lewenstein,[3,4] Jordi Tura,[5,6] and Matteo Fadel[7,†]

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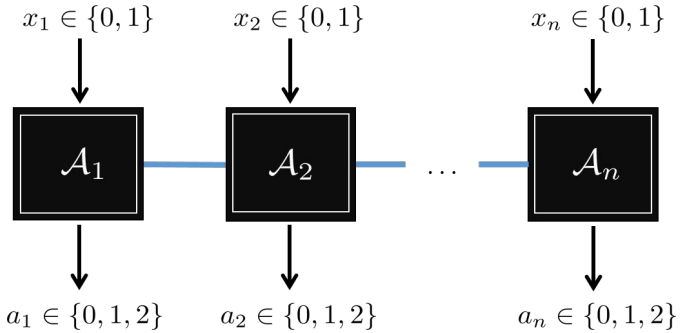
We introduce a permutationally invariant multipartite Bell inequality for many-body three-level systems and use it to investigate a connection between Bell nonlocality and (lack of) quantum chaos. Interpreting the Bell operator, defined by fixing measurement settings, as an effective Hamiltonian, we analyze its spectral statistics across different SU(3) irreducible representations and measurement choices. Surprisingly, we find that, in every irreducible representation exhibiting nonlocality, the measurement settings yielding maximal violation result in a Bell operator with Poissonian level statistics, thus signaling integrable behavior. This integrability is both unique and fragile, since generic or slightly perturbed measurements lead to the Wigner-Dyson statistics associated with chaotic behavior. Through further analysis, we are able to identify an emergent parity symmetry in the Bell operator near the point of maximal violation, providing an explanation for the observed regularity in the spectrum. These results suggest a deep interplay between optimal quantum measurements, non-local correlations, and integrability, opening new perspectives at the intersection of Bell nonlocality and quantum chaos.

Three-outcome many-body Bell inequality



PIBI

$$\mathcal{B} = \tilde{\mathcal{P}}_0 + \tilde{\mathcal{P}}_{00} - 2\tilde{\mathcal{P}}_{01} \geq 0$$

A. Aloy, GM, J. Tura, M. Fadel, Entropy **2024**, 26(10), 816

3-outcomes, arbitrary n => suited to SU3 models in the classical limit n->∞ in which chaos may arise

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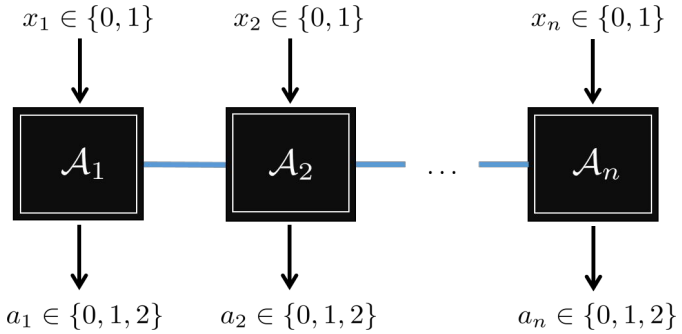

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A. Aloy, GM, J. Tura, M. Fadel, Entropy **2024**, 26(10), 816

3-outcomes, arbitrary n => suited to SU3 models in the classical limit n->∞ in which chaos may arise

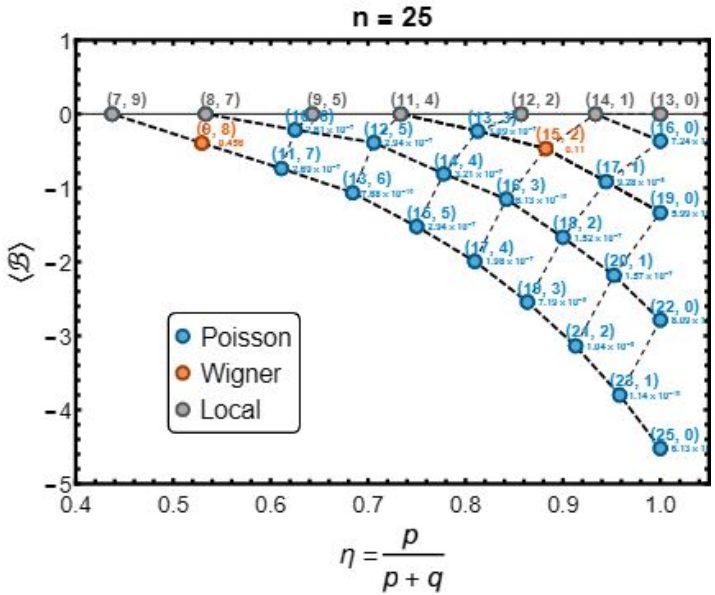
Bell operator $\mathcal{B} = \mathrm{Tr}(\hat{\mathcal{B}}\hat{\rho}), \quad \hat{\mathcal{B}} = \mathrm{polynomial}(\{\hat{\pi}_{a|x}(i)\}), \quad \hat{\pi}_{a|x}(i)$ POVMs

can we diagonalized in SU(3) irreducible representations (p,q)

We study the Bell operator eigenvalues distributions for different measurement settings. From these, we identify signatures of chaos and integrability if we interpret it as an effective Hamiltonian

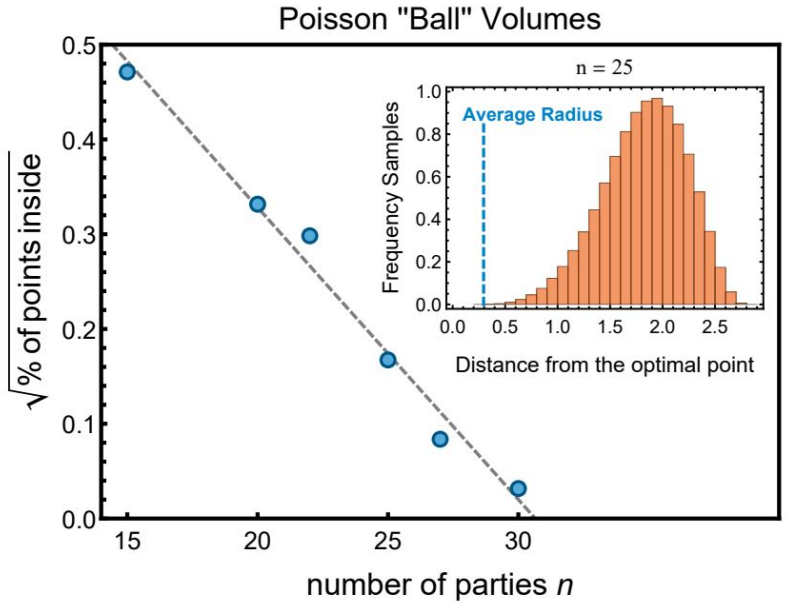
Results

For optimal measurement settings, the RCS distributions displays “Poisson behaviour”



Away from the optimal setting, the behaviour is typically Wigner

Results



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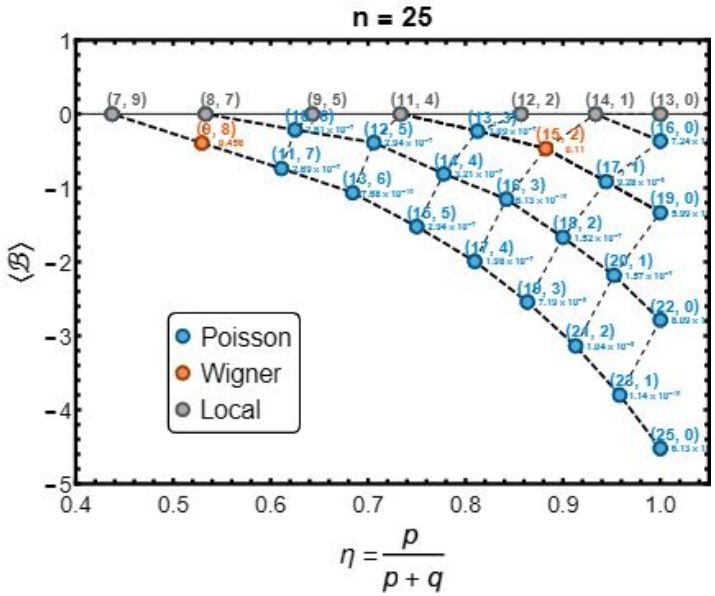

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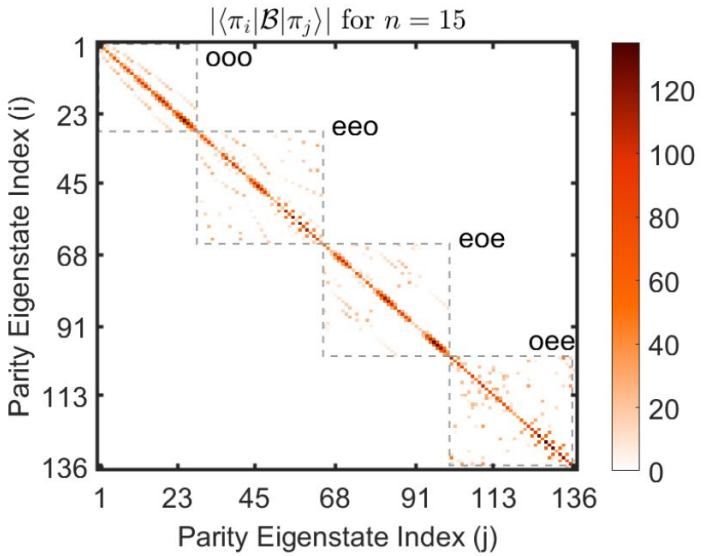
Conserved quantities

Parity sectors



Quantum correlations in space and time

Spatial Leggett-Garg Inequalities

Guillem Müller-Rigat,[1] Donato Farina,[2] Maciej Lewenstein,[1,3] and Andrea Tononi[1,4]

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(Dated: July 8, 2025)

We formulate a spatial extension of the Leggett-Garg inequality by considering three distant observers locally measuring a many-body system at three subsequent times. The spatial form, in particular, is specially suited to analyze propagation of quantum perturbations through spin chains, by capturing how a measurement at one site can later affect distant sites due to the interactions. We illustrate our proposal for a Heisenberg chain in a magnetic field, showing indeed that the first inequality-violation time scales proportiona
attribute this phenomenon to Lieb-Robinson ph
violations are anticipated when increasing the i
observable in current experiments, demonstrate
functions and the macrorealistic hypothesis. In c
a new tool for analyzing the non-relativistic dyr

4 Jul 2025

arXiv:2507.03440

Quantum Science and Technology

Celebrating the 10th anniversary of Quantum Science and Technology

LETTER

Temporal Bell inequalities in non-relativistic many-body physics

Andrea Tononi* and Maciej Lewenstein

Quantum Science and Technology, Volume 10, Number 3

Focus on Quantum Entanglement: State of the Art and Open Questions

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Conclusions

- Bell correlations are one of the strongest quantum effects that can be witnessed in many-body systems
- Classical chaos: small changes in initial conditions cause long-term unpredictability. Absence of chaos implies integrability
- Integrability/quantum chaos of the Bell operator from its spectrum distribution
- Interplay between Bell correlations and quantum chaos, but there is still much work to do!

Conclusions

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Thanks for your attention!