Krakow March 27, 2023

Reconstruction of global distributions from marginal ones with quantum computers

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Plan

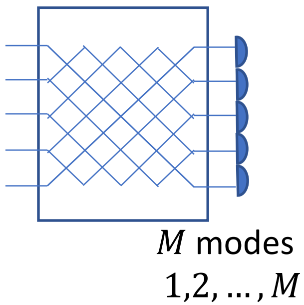
PATTERNS OF NUMBERS

COMPRESSIVE SENSING

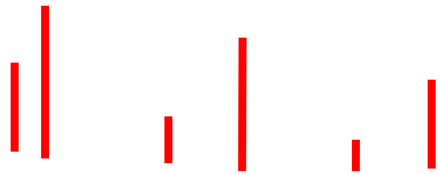
FROM CS

TO ISING

ISING SOLVERS



M – number of modes



0	0	0	0	0	0	0	0	0	0	0	0	0	0		1		2
…	…	…	…	…	…	…	…	…	…	…	…	…	…		…		…
0	0	0	0	0	0	0	0	0	1	1	1	1	1	…	0	…	2
0	0	0	1	1	1	2	2	2	0	0	0	1	1		0		2
0	1	2	0	1	2	0	1	2	0	1	2	0	1		0		2

Space of size $(N+1)^M$

$$(012)\otimes(111)^{\otimes M-1}$$

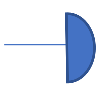
$$\ldots$$

$$(111)^{\otimes M-3}\otimes(012)\otimes(111)^{\otimes 2}$$

$$(111)^{\otimes M-2}\otimes(012)\otimes(111)$$

$$(111)^{\otimes M-1}\otimes(012)$$

Photocounters



0,1,2,…,N photons

N – maximal number of photons per mode

0s in the 1st mode

$$(100)\otimes(111)^{\otimes M-1}$$

1s in the 1st mode

$$(010)\otimes(111)^{\otimes M-1}$$

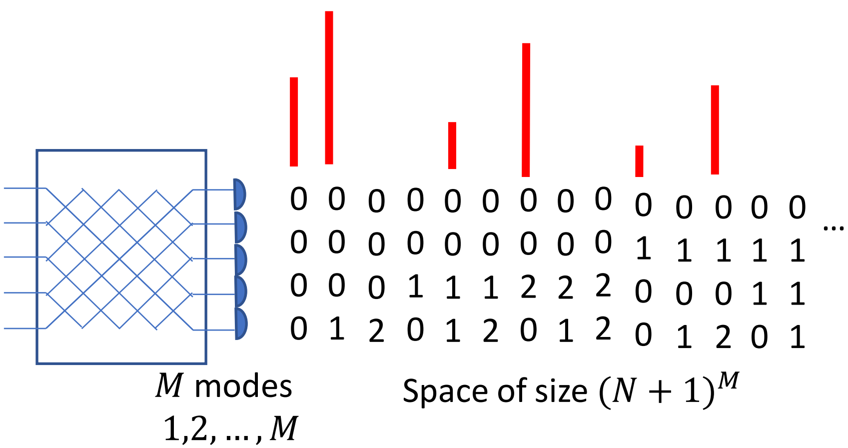
Exercise 1: 0s in the 1st mode and 1s in the 2nd

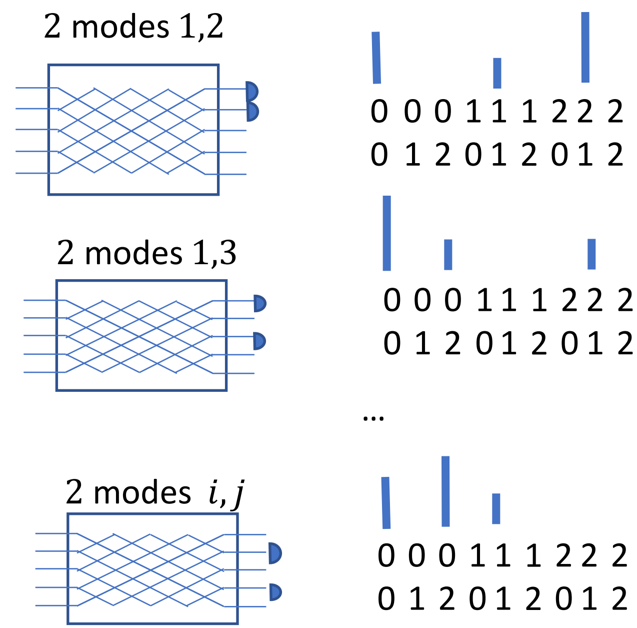
Exercise 2: 0s or 1s in the 1st mode

Exercise 3: 40th readout in our convention

Measured signal

Results of partial measurements





M – number of modes
N – maximal number of photons per mode

Measurement pattern

Mode 1,2 zero, zero $$k_{00}^{12} = (100) \otimes (100) \otimes (111)^{\otimes M-2}$$

Mode 1,2 zero, one $$k_{00}^{12} = (100) \otimes (010) \otimes (111)^{\otimes M-2}$$

Compressive Sensing

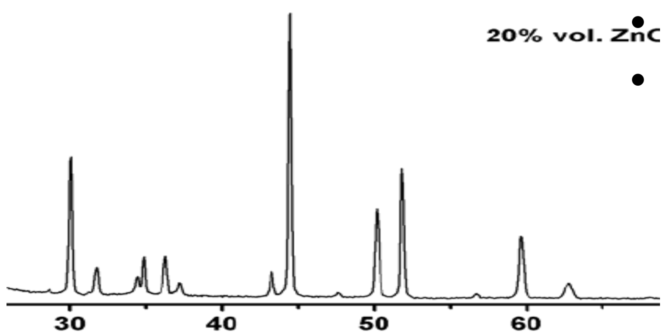
Method to reconstruct a **signal** from small number of measurements

Sparsity – small number of nonzero elements

Sparse signal

Sparse representation exists

Applications:
- Natural images
- Biomedical images
- Radar and communication signals
- Compressed speech, audio, and video



Pioneering works:
- E. Candes, IEEE Trans. IT 2005
- D. Donoho, IEEE Trans. IT 2006
- E. Candes et al., IEEE Trans. Inf. Th. 2006

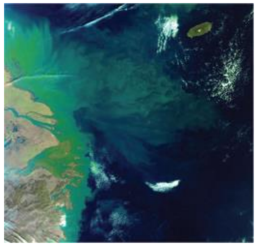
Compressive Sensing

Measurements

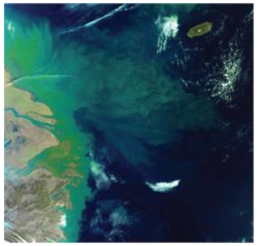
Patterns $O(s \log n/s)$

Signal

- n-dim (high)
- s-sparsity (low)



Signal reconstructed



i_1 i_2 i_3

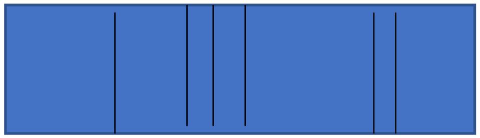
Computational Algorithm

Constrained optimization

Vector of measurements i = A matrix of masks · measured sparse signal x

- E. Candes, IEEE Trans. IT 2005
- D. Donoho, IEEE Trans. IT 2006
- E. Candes et al., IEEE Trans. Inf. Th. 2006

CS Math



If any chosen 2s columns of a rectangular matrix are linearly independent

The image of an s-sparse vector through this matrix is unique

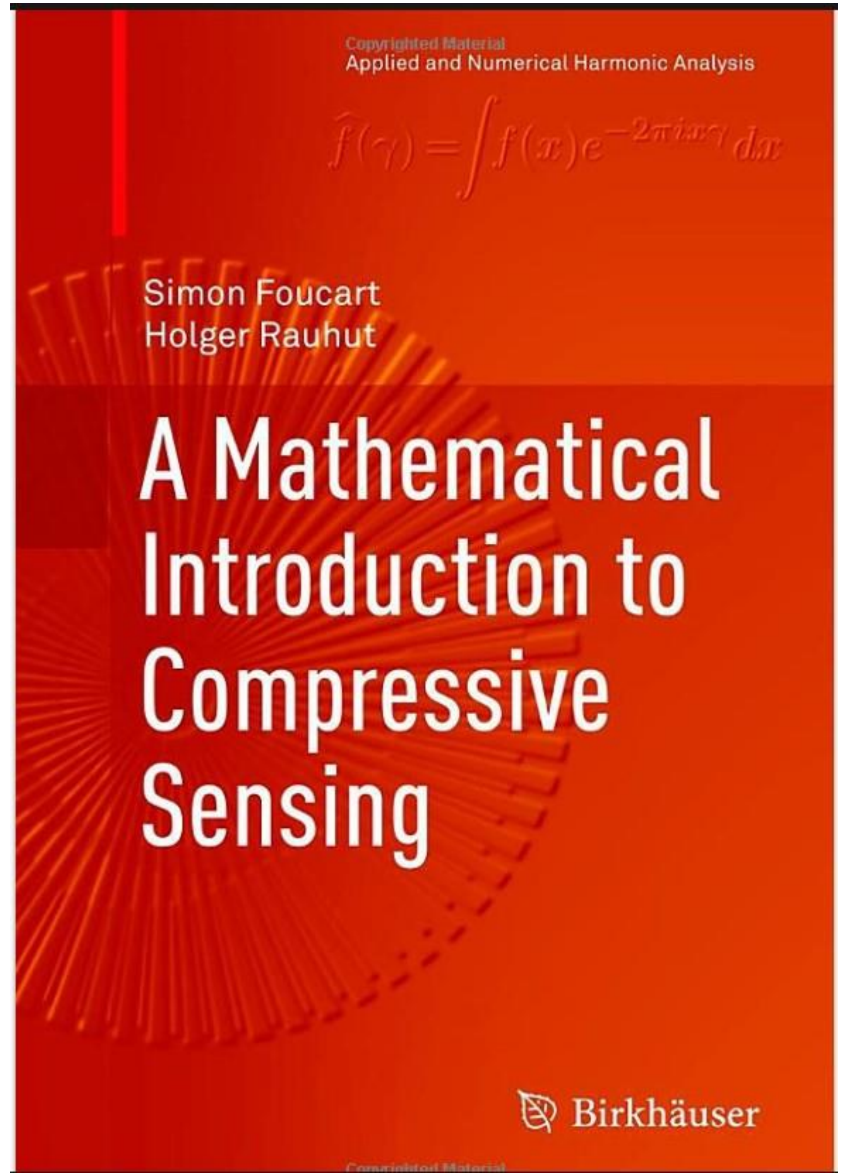
Hence, there is only one s-sparse vector fitting the image

Hence, the correct solution is the sparsest among the solutions that fit

Then find them most sparse by solving constraint optimization problem

$argmin_x \|x\|_1$ given that $Ax = y$ or $argmin_x \|y - Ax\|_2 + \lambda\left|\left|x\right|\right|_1$

BOOK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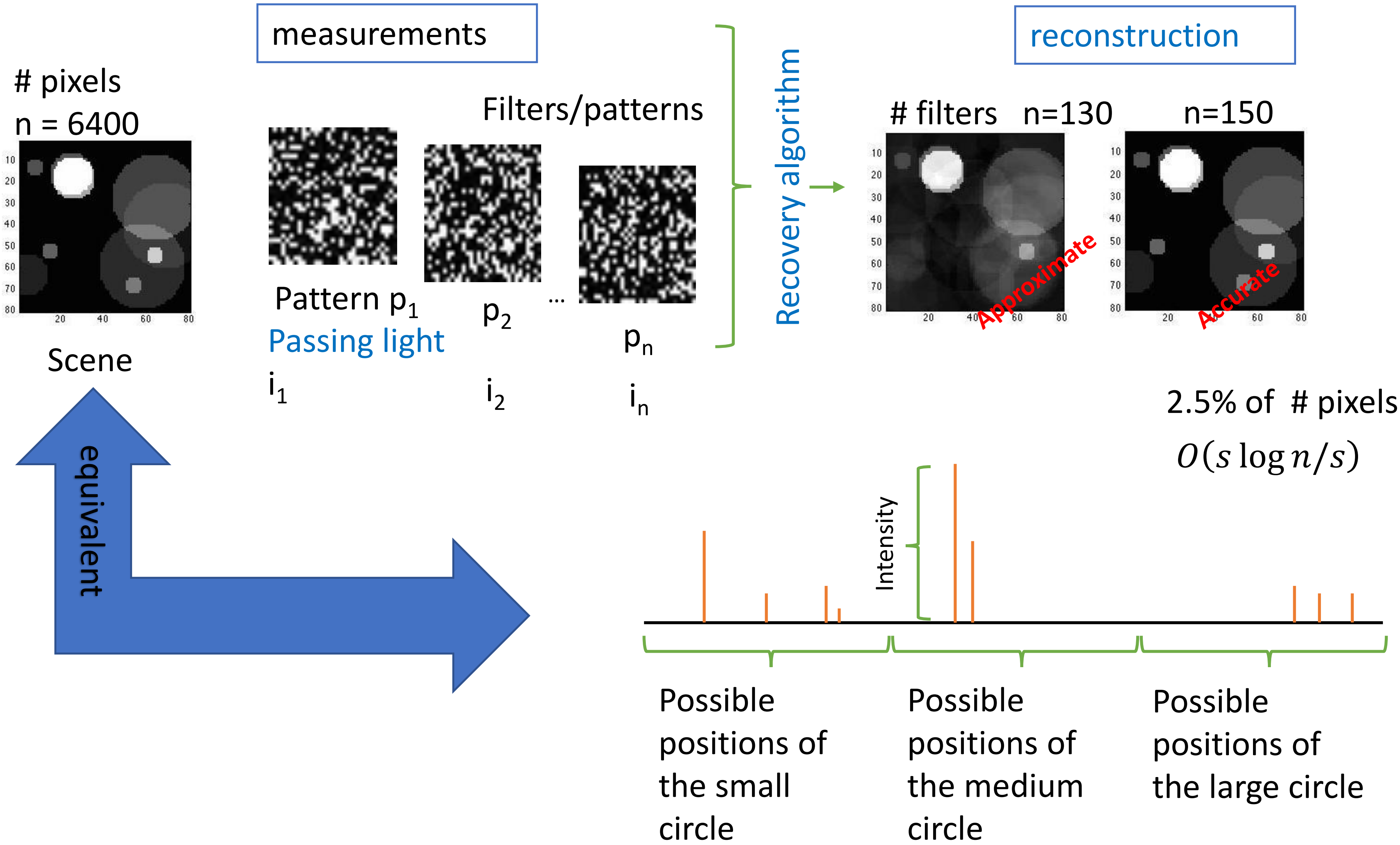
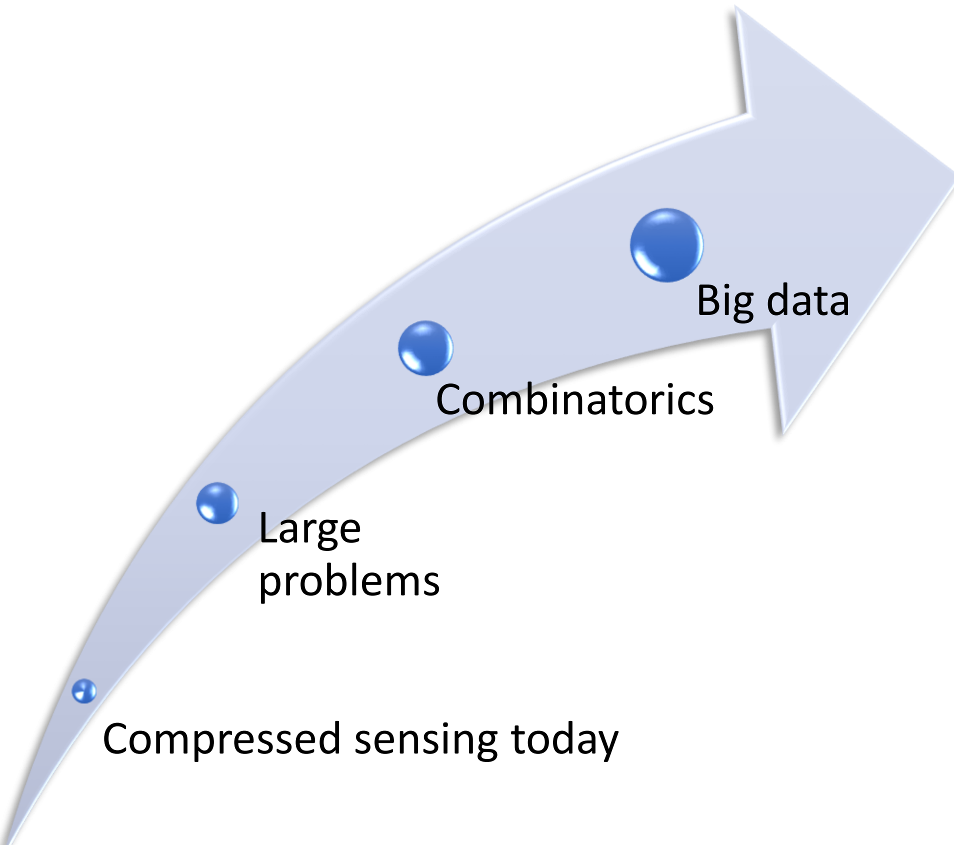
Applied and Numerical Harmonic Analysis
$\hat{f}(\gamma)=\int f(x)e^{-2\pi i x\gamma}\,dx$
Simon Foucart
Holger Rauhut
A Mathematical Introduction to Compressive Sensing
Birkhäuser

Example



measurements
# pixels
n = 6400
Scene
Filters/patterns
Pattern p_1
Passing light
p_2
…
p_n
i_1
i_2
i_n
Recovery algorithm
reconstruction
# filters n=130
n=150
Approximate
Accurate
2.5% of # pixels
$O(s \log n/s)$
equivalent
Intensity
Possible positions of the small circle
Possible positions of the medium circle
Possible positions of the large circle

Future



- Large scale
- Memory issue
- Linear algebra inefficient
- No exact optimization techniques

[V. Cevher, et al., IEEE Sig. Proc. Mag. 2014
Convex Optimization for Big Data]

Structured patterns

First order methods as reconstruction algorithms

- Small scale
- No memory issue
- Linear algebra efficient (matrix inverse, matrix multiplication, ..)
- Optimization techniques, exact, fast

Structured patterns → **Ising type**

$$(11) \otimes (11) \otimes (10) \otimes (01) \otimes (11)$$
$$(11) \otimes (11) \otimes (01) \otimes (01) \otimes (11)$$

First order method → **Matching pursuit** [Mallat et al. 1993]

Notice that each of these patterns can be decomposed into

$$(11) \otimes (11) \otimes diag(\sigma_z) \otimes diag(\sigma_z) \otimes (11)$$
$$(11) \otimes (11) \otimes diag(\sigma_z) \otimes (11) \otimes (11)$$
$$(11) \otimes (11) \otimes (11) \otimes diag(\sigma_z) \otimes (11)$$
$$(11) \otimes (11) \otimes (11) \otimes (11) \otimes (11)$$

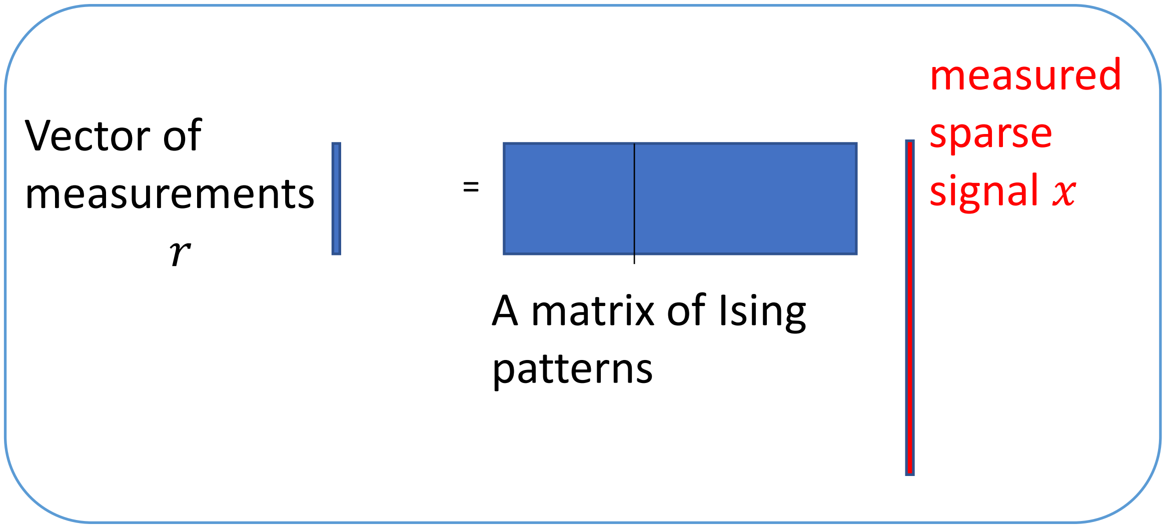
$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$H = \sum_{i,j} J_{ij}\, {\sigma_z}^i {\sigma_z}^j + \sum_{i,j} k_i\, {\sigma_z}^i$$

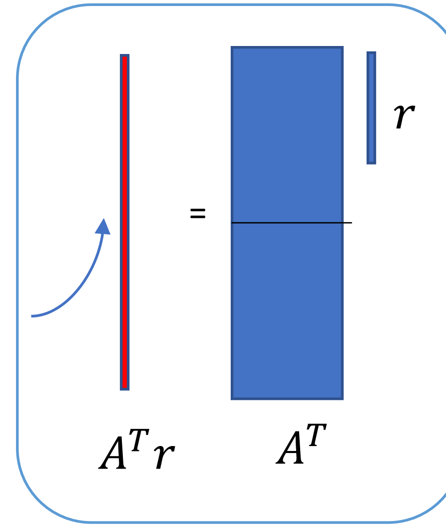
W. Roga & M. Takeoka, Sci. Rep. 2020
Classical simulation of boson sampling with sparse output

HOW WE FIND ISING
1/2

Matching Pursuit



Localize maximum



- Support detection
  - Find a column of A that is the closest to the current residue, $max\ |A^T r_{i-1}|$
- Update r
- Repeat

With the chosen patterns this problem is the Ising problem

Set of measurements and constrained optimization method depend on the sparsity s

s – number of nonzero elements in the signal

	s - small	**s - larger**	**s – large**
Measurement:	Nearest neighbor Ising patterns	All pairs Ising patterns	Beyond Ising : “Three spins” Ising patterns
Constrained optimization:	Classically efficient	Quantum/simulated annealer	QAOA

QAOA – Quantum Approximate Optimization Algorithm

QUANTUM ANNEALER Adiabatic quantum computation (AQC)

$$H(t) = \left(1 - \frac{t}{T}\right) H_B + \left(\frac{t}{T}\right) H_P$$

Initial Hamiltonian H_B

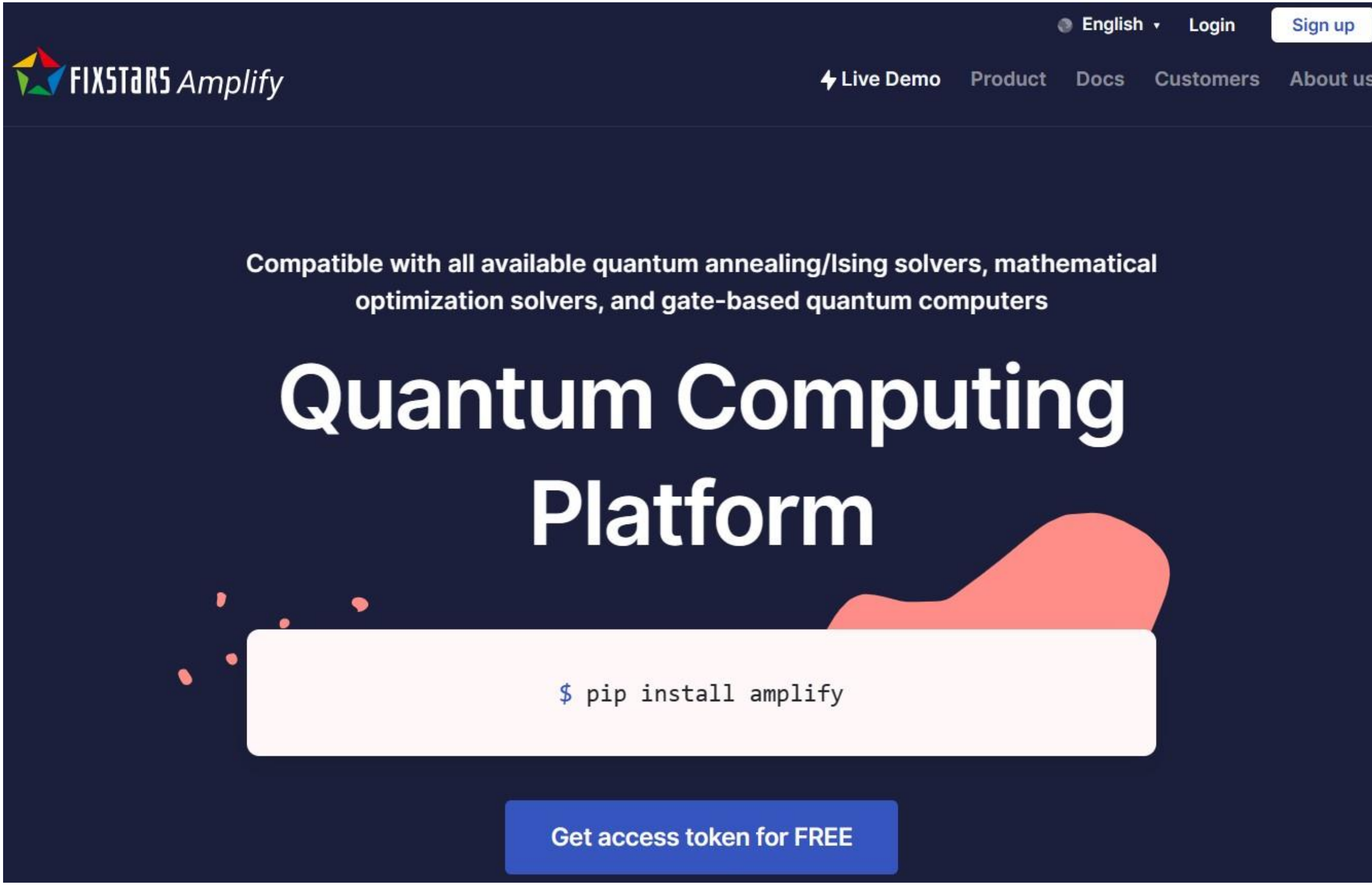
Problem Hamiltonian H_P

Adiabatic theorem

$$T \sim \frac{\epsilon}{g_{min}^2}$$

$$g_{min} = \min E_1 - E_0$$

Minimum gap between the ground and excited sate during all evolution



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Fixstars

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Hitachi, Ltd.

CMOS Annealing Machine

Gate-based Quantum Computer

IBM

IBM

IBM Quantum

Quantum Circuit Simulator

Qulacs

Qulacs

Summary: Compressive sensing for large scale problems

Measurements

- Structured constraints
- Ising patterns

Optimization algorithm

- First order method
- Matching pursuit
- Classical Ising problem

Solver

- Depending on the sparsity
- Classical
- Quantum annealers
- QAOA

- Thank you for your attention