Flexible quantum state tomography

Daniel Uzcátegui, Gabriel Senno, Dardo Goyeneche

QUANTUM INFORMATION GROUP ANTOFAGASTA

March 25, 2020 - Kraków, Poland

Motivation

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There is no explicit formula for Quantum State tomography from a given set of quantum measurements

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Numerical computations are required in general (very demanding in high dimensions)

Previous Works

PHYSICAL REVIEW A
covering atomic, molecular, and optical physics and quantum information

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State determination: An iterative algorithm

Dardo M. Goyeneche and Alberto C. de la Torre
Phys. Rev. A **77**, 042116 – Published 23 April 2008

Quantum tomography meets dynamical systems and bifurcations theory

Journal of Mathematical Physics **55**, 062103 (2014); https://doi.org/10.1063/1.4881855

D. Goyeneche[1, a)] *and* A. C. de la Torre[2]

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Impose physical information in a quantum state

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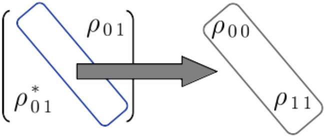
1) Remove the main diagonal



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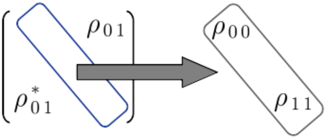
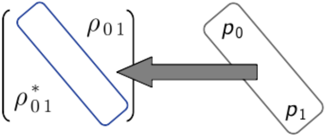


2) Impose probabilities



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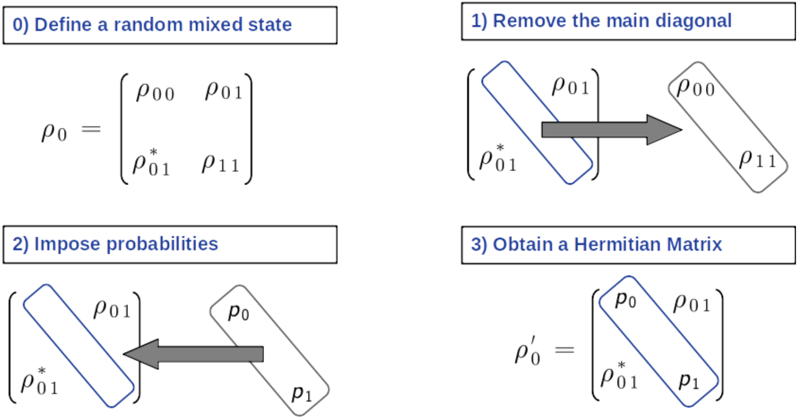

The Physical Imposition Operator (PIO)

Steps 1 to 3 define the Physical Imposition Operator, denoted as $T_{A_1}(\;_0)$. When considering two incompatible observables $A_1; A_2$, we have the composite operator:

$$T_{A_1 A_2}(\;_0) = T_{A_2} \quad T_{A_1}(\;_0) = T_{A_2}(\;^{0}_{0})$$

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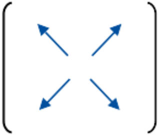

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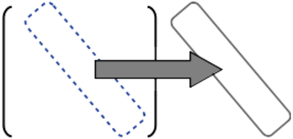

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Remove information

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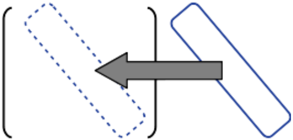

Rotate

Impose information about A_2

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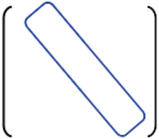

Rotate

Imposed information

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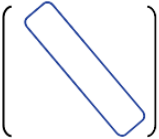
$U^{y}_{A_2}\ {}^0_0 U_{A_2}$!  ! $U_{A_2}(\quad)U^{y}_{A_2}$

Rotate — Imposed information — Reverse Rotation

The Physical Imposition Operator (PIO)

For m observables $A_1; : : : ; A_m$, the PIO is defined as:

$$T_{A_1 ::: A_m} = T_{A_m} \qquad\qquad T_{A_1}$$

The solution to the quantum state tomography problem is given by the limit of the following sequence:

$$_n = (T_{A_1 ::: A_m})^n(\;_0)$$

Bloch sphere representation of PIO

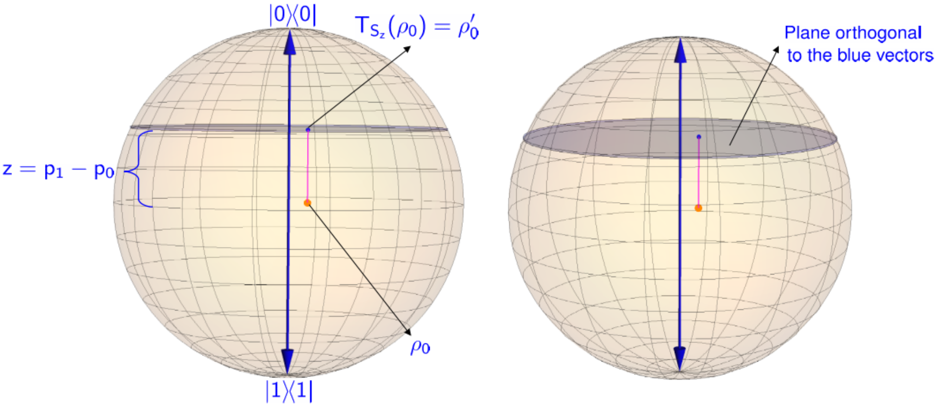


Figure 10: Action of the PIO in the Bloch sphere: orthogonal projection to a disk.

Bloch sphere representation

Figure 11: Convergence of the PIO when considering *two* observables.

Convergence

We consider the following metrics:

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The Hellinger Distance:

$$H(\mathbf{p},\mathbf{q}) = \left(2 - 2 \sum_{i=0}^{d-1} \sqrt{p_i}\sqrt{q_i} \right)^{1/2} \quad (1)$$

where $\mathbf{p} = \{p_0, \ldots, p_{d-1}\}$, $\mathbf{q} = \{q_0, \ldots, q_{d-1}\}$ and $q_i = Tr \quad _n \quad _i \quad _i$

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The Hellinger Distance:

$$H(p, q) = \left(\frac{1}{2} \sum_{i=0}^{d-1} \left(\sqrt{p_i} - \sqrt{q_i} \right)^2 \right)^{1/2} \qquad (1)$$

where $p = \{p_0, \ldots, p_{d-1}\}$, $q = \{q_0, \ldots, q_{d-1}\}$ and $q_i = \mathrm{Tr}\ {}_n\ {}_i\ {}_i$

The Distributional Distance:

$$D_{A_1 \cdots A_m}(P, Q) = \sqrt{\frac{1}{m} \sum_{k=1}^{m} H(p^k, q^k)^2} \qquad (2)$$

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The Hilbert-Schmidt Distance:

$$HSD(\ _{gen};\ _{n}) = \sqrt{\mathrm{Tr}[(\ _{gen} - \ _{n})^2]} \quad (3)$$

where $\ _{gen}$ is the so-called generator state.

Bloch sphere representation

Figure 12: Convergence of the PIO when considering TWO incompatible observables.

Bloch sphere representation

Figure 13: Convergence of the PIO for THREE incompatible observables.

Bloch sphere representation

Figure 14: Convergence of the PIO for THREE incompatible observables.

Mutually Unbiased Bases (MUB)

Two orthonormal bases $\left\{ {}_0 ; \ldots ; {}_{d-1} \right\}$ and $\left\{ \left| {}_0 \right\rangle ; \ldots ; \left| {}_{d-1} \right\rangle \right\}$, are MUB if:

$$\left| \left\langle {}_i \middle| {}_j \right\rangle \right|^2 = \frac{1}{d} \quad \forall\, i ; j = 0 ; \ldots ; d - 1 \tag{4}$$

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In prime power dimension d there are maximal sets of $d + 1$ MUB [1].

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In prime power dimension d there are maximal sets of $d+1$ MUB [1]. Any density matrix can be univocally determined from maximal sets of MUB:

$$= \sum_{k=1}^{d+1} \sum_{j=0}^{d-1} p_j^k \left|\ _j^k\right\rangle\left\langle\ _j^k\right| - 1 \tag{5}$$

where $p_j^k = p(\ _j^k)$ is the probability to obtain the result $\ _j^k$.

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Mutually Unbiased Bases (MUB)

Figure 15: Algorithm converges in A SINGLE iteration for complementary observables.

Physical imposition operator as a quantum channel?

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(a) Projections can be non-physical.

Physical imposition operator as a quantum channel?

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(b) Iterations eventually produce a physical state

Kaczmarz Method (Stefan Kaczmarz, 1937)[2]

A system of linear equations $Ax = b$ can be solved applying the following iterations:

$$x_{n+1} = x_n + \frac{(b_i - x_n^T a_i)}{|a_i|^2} a_i$$

For instance,

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

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Our method vs. Super-fast maximum likelihood [3]:

[3] J. Shang, Z. Zhang, and H. K. Ng, Superfast maximum-likelihood reconstruction for quantum tomography, Phys. Rev. A 95, 062336 (2017).

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Figure 17: Fidelity comparison for PIO and Super-fast MLE. Average runtime over 100 trials by using 3^n tensor product Pauli bases, with n being the number of qubits. We consider 1000 copies of the state (nite statistics).

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Figure 19: Fidelity comparison for PIO and Super-fast MLE. Average runtime over 100 trials by using maximal sets of MUB measurements for n qubits. We consider 1000 copies of the state (nite statistics).

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Figure 20: Average runtime over 100 trials by using maximal sets of MUB for n qubits. We consider 1000 copies of the state (nite statistics).

[6] J. Shang, Z. Zhang, and H. K. Ng, Superfast maximum-likelihood reconstruction for quantum tomography, Phys. Rev. A 95, 062336 (2017).

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- It works for any set of quantum measurements.
- Maximal advantage is obtained for complementary observables.
- It has a `decent' behavior when compared with ultra-fast maximum likelihood state estimation.
- A modified version of the physical imposition operator defines a physical operation.

Open issues

Physical implementation

Open issues

Physical implementation

Multipartite quantum state tomography

Open issues

Physical implementation

Multipartite quantum state tomography

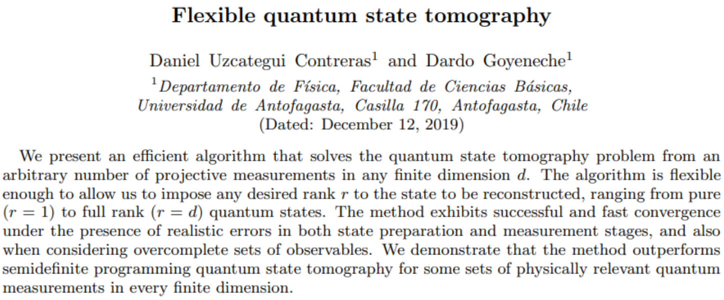
POVM measurements

Acknowledgements:

Grant FONDECYT Iniciación number 11180474, Chile

Project ANT1856, Universidad de Antofagasta

In progress..


Flexible quantum state tomography

Daniel Uzcategui Contreras[1] and Dardo Goyeneche[1]
[1] *Departamento de Física, Facultad de Ciencias Básicas, Universidad de Antofagasta, Casilla 170, Antofagasta, Chile*
(Dated: December 12, 2019)

We present an efficient algorithm that solves the quantum state tomography problem from an arbitrary number of projective measurements in any finite dimension d. The algorithm is flexible enough to allow us to impose any desired rank r to the state to be reconstructed, ranging from pure ($r = 1$) to full rank ($r = d$) quantum states. The method exhibits successful and fast convergence under the presence of realistic errors in both state preparation and measurement stages, and also when considering overcomplete sets of observables. We demonstrate that the method outperforms semidefinite programming quantum state tomography for some sets of physically relevant quantum measurements in every finite dimension.

https://arxiv.org/abs/1912.05754

Quantum toilets

Many thanks for your attention!