How Quantum Is a “Quantum Walk”?

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Review

Classical Random walk (RW)

"p": for "HEADS" and walking to "RIGHT"

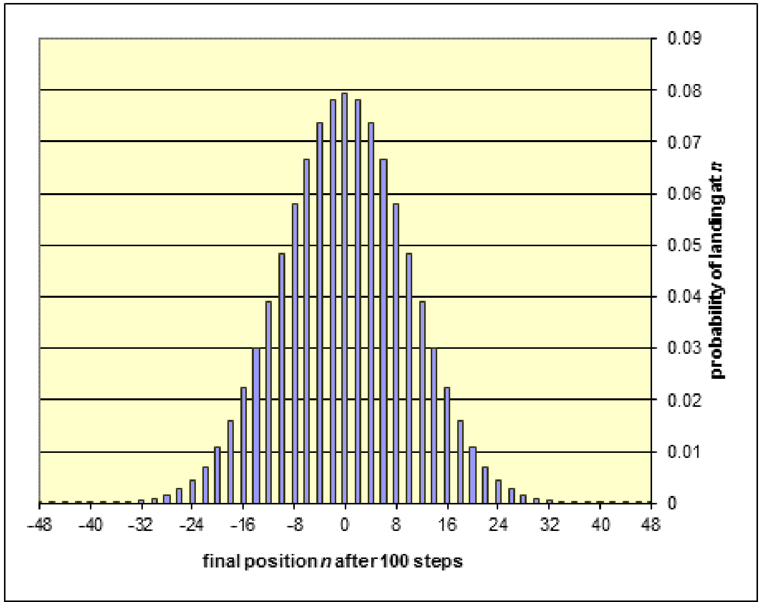
"1 – p": for "TAILS" and walking to "LEFT"

"t": number of steps

Review (RW)

$$\mathbf{P_{RW}(x = 2n - t, t) = \frac{t!}{n!(t-n)!} p^n (1-p)^{t-n}}$$

$$\mathbf{P_{RW}(2n - t, t \gg 1/p) = \frac{1}{\sqrt{2\pi t p(1-p)}} \exp\left(-\frac{(n-pt)^2}{2tp(1-p)}\right)}$$



$$\langle \mathbf{x} \rangle = (\mathbf{2p} - \mathbf{1})\mathbf{t}$$

$$\sigma_{\mathbf{x}}^{\mathbf{2}} = 4\mathbf{p}(1 - \mathbf{p})\mathbf{t}$$

Quantum Random Walk (QW)

$\mathbf{t}$

$$\mathbf{U} = \left(\sum_{\mathbf{x}} |\mathbf{R}\rangle\langle\mathbf{R}| \otimes |\mathbf{x}+\mathbf{1}\rangle\langle\mathbf{x}| + |\mathbf{L}\rangle\langle\mathbf{L}| \otimes |\mathbf{x}-\mathbf{1}\rangle\langle\mathbf{x}|\right)(\mathbf{V} \otimes \mathbb{I})$$

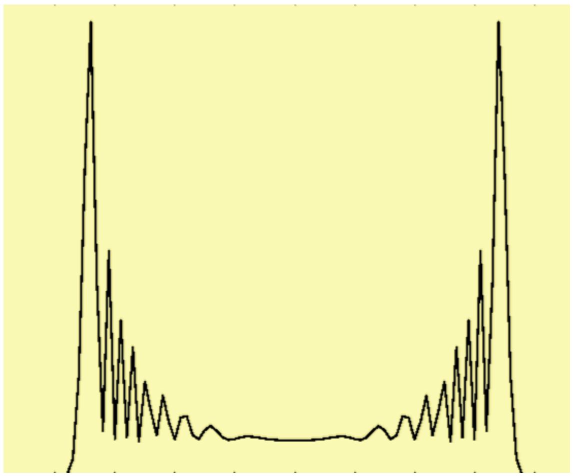
$$|\mathrm{coin}\rangle \otimes |\mathrm{walker}\rangle = \left(\sqrt{\mathrm{p}}|\mathrm{R}\rangle + \mathrm{e}^{\mathrm{i}\alpha}\sqrt{1-\mathrm{p}}|\mathrm{L}\rangle\right) \otimes |0\rangle$$

$$\varrho_{\mathbf{cw}}(\mathbf{t})$$

$$\mathrm{P}_{\mathrm{QW}}(\mathrm{x},\mathrm{t}) = \mathrm{Tr}\left((\mathbb{I} \otimes |\mathrm{x}\rangle\langle\mathrm{x}|)\,\varrho_{\mathbf{cw}}(\mathbf{t})\right)$$

Quantum Random Walk (QW)

$$P_{QW}(x,t) = \mathrm{Tr}\Big(\big(\mathbb{I} \otimes |x\rangle\langle x|\big)\varrho_{cw}(t)\Big)$$



Noiseless and Symmetric Hadamard Coin

$$\langle x \rangle \propto t$$

$$\sigma_x^2 \propto t^2$$

Quantum Random Walk (QW)

$\mathbf{t}$

Noise Operation

$$\mathbf{U} = \left(\sum_{\mathbf{x}} |\mathbf{R}\rangle\langle\mathbf{R}| \otimes |\mathbf{x}+\mathbf{1}\rangle\langle\mathbf{x}| + |\mathbf{L}\rangle\langle\mathbf{L}| \otimes |\mathbf{x}-\mathbf{1}\rangle\langle\mathbf{x}|\right)(\mathbf{V} \otimes \mathbb{I})$$

$$\langle \mathbf{x} \rangle \propto \mathbf{t}$$

$$\sigma_{\mathbf{x}}^{\mathbf{2}} \propto \mathbf{t}^{\mathbf{2}}$$

***Unital decaying noise, such as decoherence, results in linear variance**

Quantumness:

$$\mathbf{Q(t) = \min_{P_{RW}} D(P_{QW} \| P_{RW})}$$

- $\mathbf{D}$ is "relative entropy"
- The optimized RW is the one with: $\langle \mathbf{x} \rangle_{\mathbf{RW}} = \langle \mathbf{x} \rangle_{\mathbf{QW}}$
- $\mathbf{Q(t) = H(P_{RW}(t)) - H(P_{QW}(t)) + \log e \frac{\sigma^2_{QW}(t) - \sigma^2_{RW}(t)}{2\sigma^2_{RW}(t)}, t \gg 1}$

Total Quantumness:

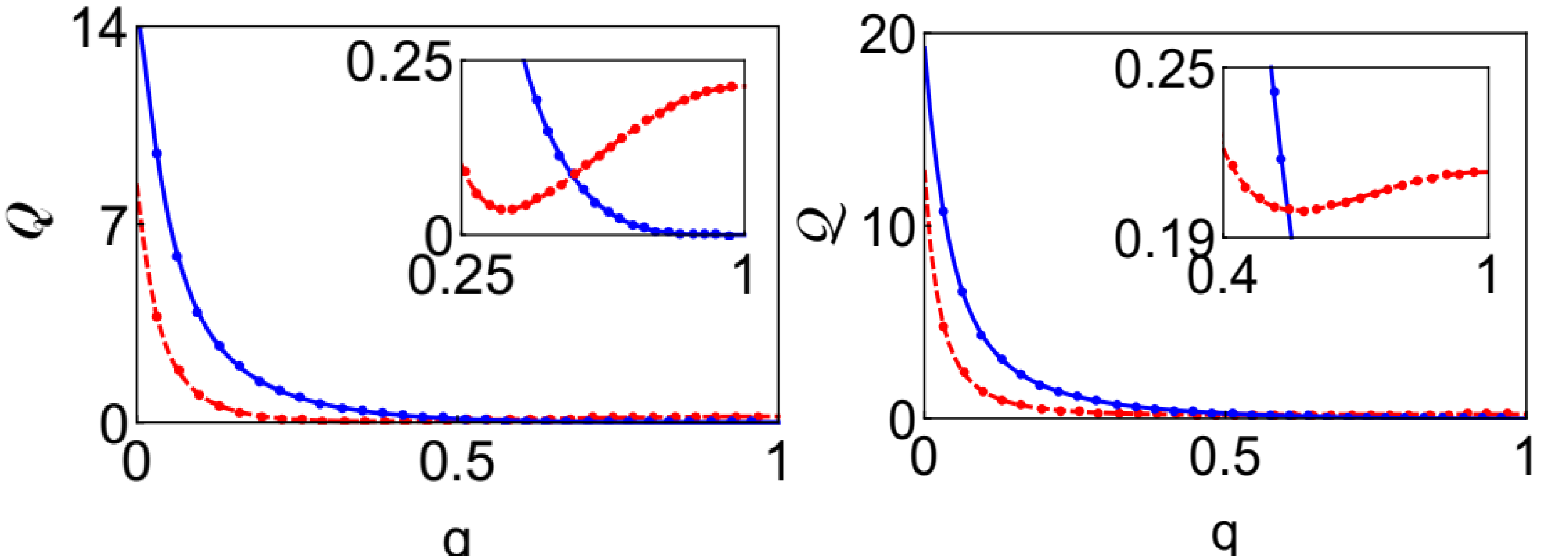
$$\mathcal{Q}(\mathbf{t}) = \min_{\rho_{\mathbf{RW}}} \mathbf{D}(\rho_{\mathbf{QW}} \| \rho_{\mathbf{RW}})$$

- $\rho_{\mathbf{QW}} = \mathbf{Tr_c}(\varrho_{\mathbf{cw}}(\mathbf{t}))$
- $\rho_{\mathbf{RW}}$ is gained from QW protocol using a special contraction noise on the coin.
- The optimized RW is the same as the one of $\mathbf{Q(t)}$
- $\mathcal{Q}(\mathbf{t}) = \mathbf{Q(t)} + \mathbf{C}(\rho_{\mathbf{QW}}(\mathbf{t}))$

  where $\mathbf{C}(\rho_{\mathbf{QW}}(\mathbf{t}))$ is quantum coherence of $\rho_{\mathbf{QW}}$

Quantum Random Walk (QW)

Example: Q, $\mathcal{Q}$ and $C(\rho_{QW})$ under decoherence on the coin for $\mathbf{V} = \begin{pmatrix} 0.54 & 0.84 \\ 0.84 & -0.54 \end{pmatrix}$, $\mathbf{V} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, and $|\mathbf{coin}\rangle = |\mathbf{R}\rangle$



Quantum Random Walk (QW)

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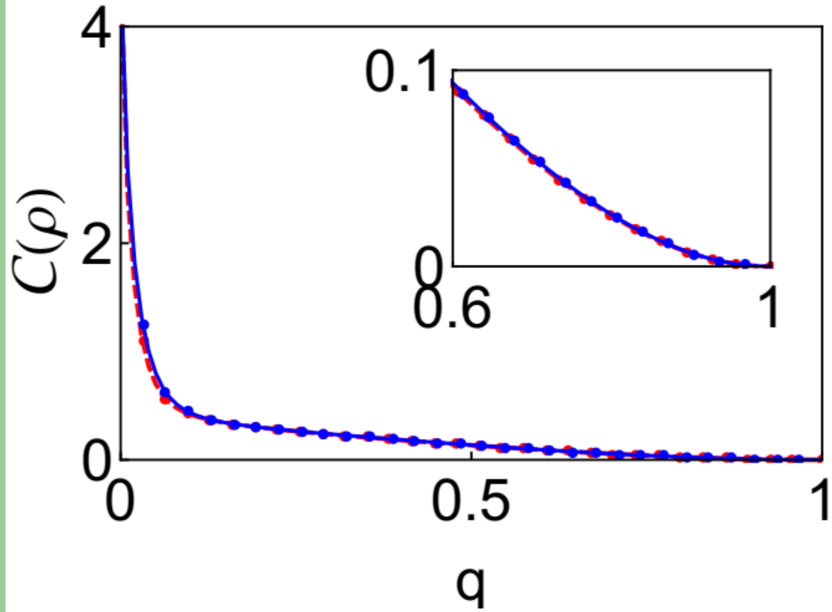
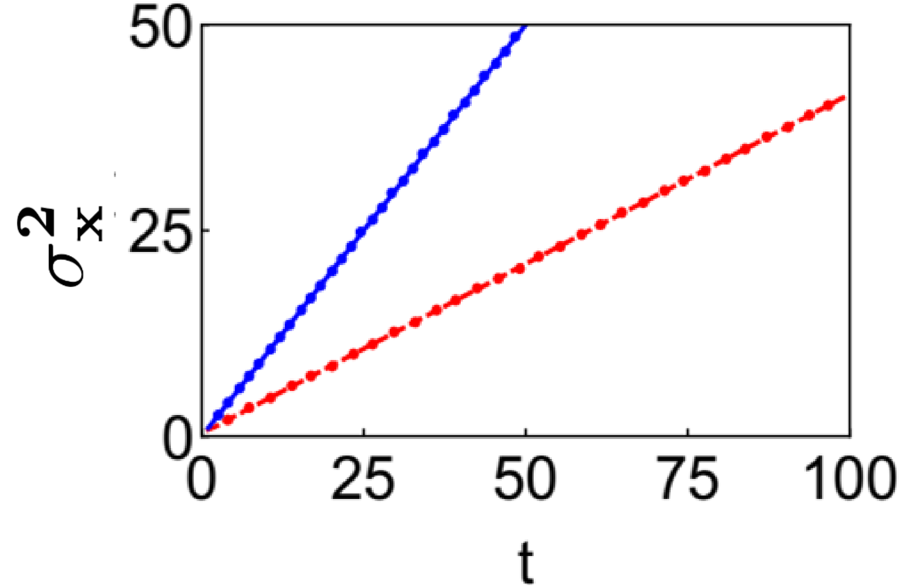
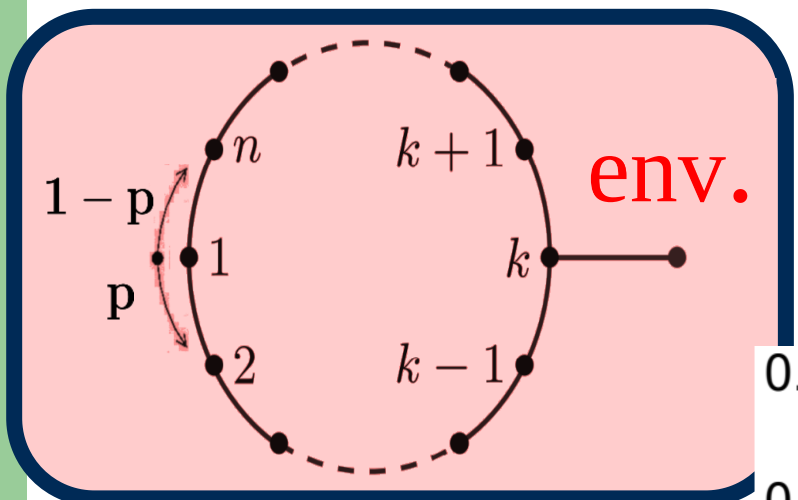


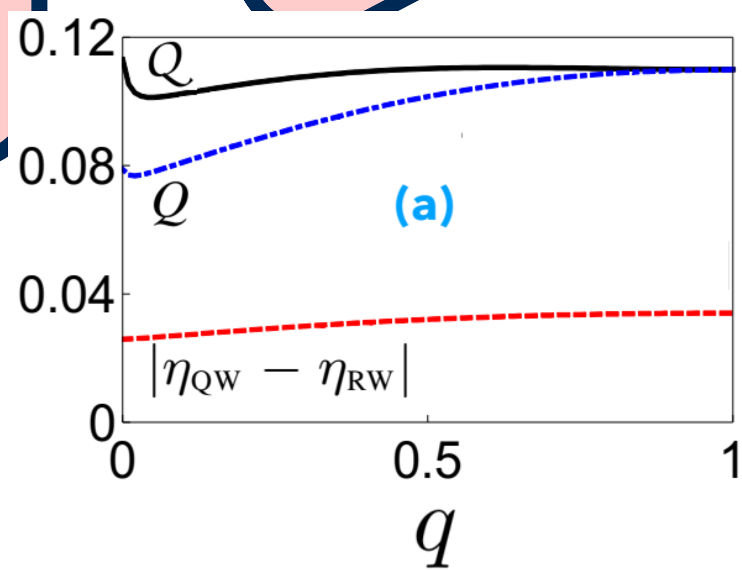


Quantum Transport

$\mathcal{Q} \geq \mathbf{Q} \geq u \geq \mathbf{0}$

u is deviation of **quantum** Transport efficiency from classical transport, to env.





Thank you