The uncertainty principle ***does not entirely*** determine non-locality in quantum mechanics

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UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE

Introduction

- Uncertainty relations
- Quantum steering
- No-signalling principle

Uncertainty relations in quantum mechanics

$\Delta x \Delta p \geq \frac{\hbar}{2}$ (Heisenberg)

$\Delta A \Delta B \geq \left| \frac{1}{2} \langle \{ \hat{A}, \hat{B} \} \rangle - \langle \hat{A} \rangle \langle \hat{B} \rangle \right|^2 + \left| \frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right|^2$ (Schrödinger)

Entropic uncertainty relations

$$H^{(a)} + H^{(b)} \geq -2\ln\left[\frac{1}{2}(1 + C_B)\right]$$ (Deutsch)

$$H^{(a)} + H^{(b)} \geq -2\ln C_B$$ (Maassen-Uffink)

$$H = -Tr[\rho\, Log(\rho)]$$

$$C_B = \sup_{(i,j)} |\langle a_i | b_j \rangle|$$

D. Deutsch, Phys. Rev. Lett. 50, 631 (1983)
H. Maassen, J. Uffink, Phys. Rev. Lett. 60, 1103 (1988)

Fine grained uncertainty relations

$$\sum_{x=1}^{n} \sum_{a=1}^{m} V(a,x)\, p(x)\, p(a|x)_{\sigma} \leq C$$

J. Oppenheim, S. Wehner, Science, 330, 6007, 1072-1074 (2010)

Fine grained uncertainty relations

- An example -

$$\frac{1}{2}p(a|Z) + \frac{1}{2}p(b|X) \leq \frac{1}{2} + \frac{1}{2\sqrt{2}} \approx 0.8535$$

Fine grained uncertainty relations

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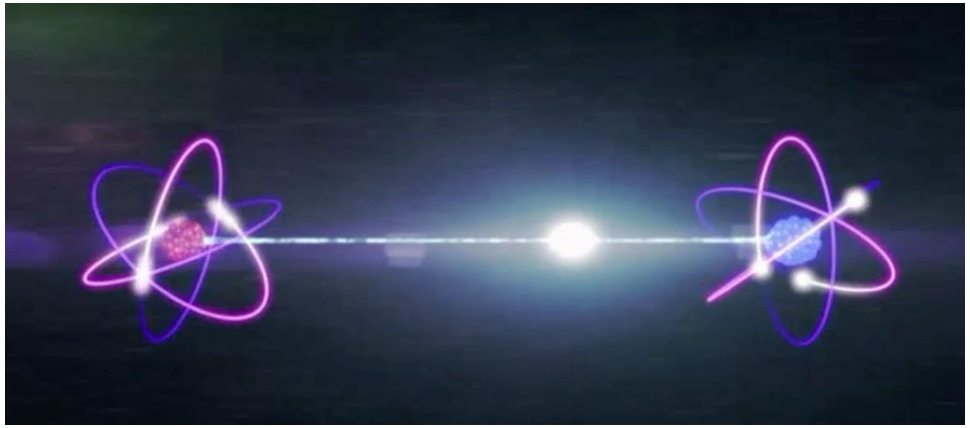
The inequality is not saturated by eigenstates of observables:

$$p(a|Z) = 1 \qquad p(b|X) = 1/2$$

Quantum steering

$$\sigma_{a|x}^{(B)} = \sum_{\lambda} q(a|x, \lambda)\rho_{\lambda}$$

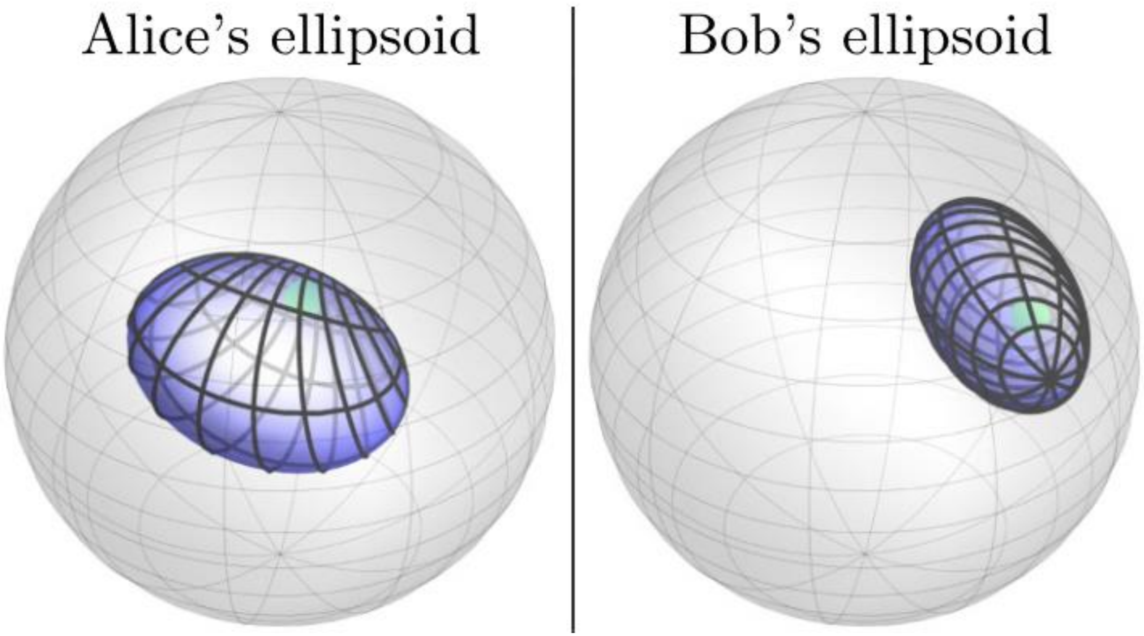
$$q(a|x, \lambda) := p(\lambda)p(a|x, \lambda)$$



If Bob reduction can be written in this form for any possible measurement then the state it allows a Local Hiden State (LHS) decomposition.

If the state cannot be written in such form then <u>Alice can steer the state of Bob.</u>

Quantum steering for two qubits



S. Jevtic, M. Pusey, D. Jennings, T. Rudolph, Quantum Steering Ellipsoids, Phys. Rev. Lett. 113, 020402 (2014)

No-signalling principle

Information cannot travel faster than light…

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$$\hat{\sigma}_B = \sum_a p(a|x)\hat{\sigma}^B_{a|x}$$

(If Bob **doesn't have communication** with Alice)

e.g. $|\phi_{AB}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ $\qquad \sigma_B = \frac{1}{2}\mathbb{I}$

Bell inequalities

- Quantum games
- Conjecture
- Counterexample

Bell inequalities (as a quantum game)

INPUT	x	y
	ALICE	**BOB**
OUTPUT	a	b

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3) *They win the game if* $ab = x \oplus y$ *for every* a, b, x, y

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$$\sum \pi_{AB}(x, y)\ V(a,b,x,y)\ P(ab|xy) \leq \omega(g)$$

CHSH inequality:

¼[P(00|00)+ P(01|00)+ P(10|00)+ P(11|01)+
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ab	$x \oplus y$
00	0
01	0
10	0
11	1

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Probability to win the game:

Classical: $3/4 = 0.75$

Quantum: $1/2 + 1/4\sqrt{2} \approx 0.85$

No-signalling theory: 1

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Degree of non-locality in quantum mechanics = Strength of the uncertainty principle (local) + Degre[illegible]teering

CONJECTURE

J. Oppenheim and S. Wehner, Science, Vol. 330, No. 6007, 1072 (2010).

The uncertainty principle **determines** the strength of non-locality in QM (conjecture)

$$\sum_{\substack{x\in\mathcal{X}\\ y\in\mathcal{Y}}} \pi_{AB}(x,y) \sum_{\substack{a\in\mathcal{A}\\ b\in\mathcal{B}}} V(a,b|x,y)P(a,b|x,y) \leq \omega(g)$$

$$P(a,b|x,y) = \mathrm{Tr}\left(\rho M_a^x \otimes M_b^y\right)$$

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$$\sum_{\substack{x\in\mathcal{X}\\ a\in\mathcal{A}}} \pi_A(x)P(a|x) \sum_{\substack{y\in\mathcal{Y}\\ b\in\mathcal{B}}} \pi_B(y|x)V(a,b|x,y)P(b|y,x,a)$$

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$$\sum_{\substack{x\in\mathcal{X}\\ y\in\mathcal{Y}}} \pi_{AB}(x,y) \sum_{\substack{a\in\mathcal{A}\\ b\in\mathcal{B}}} V(a,b|x,y)P(a,b|x,y) \leq \omega(g) \qquad P(a,b|x,y) = \mathrm{Tr}\left(\rho M_a^x \otimes M_b^y\right)$$

$$\sum_{\substack{x\in\mathcal{X}\\ a\in\mathcal{A}}} \pi_A(x)P(a|x) \sum_{\substack{y\in\mathcal{Y}\\ b\in\mathcal{B}}} \pi_B(y|x)V(a,b|x,y)P(b|y,x,a)$$

Let us consider the *free game* scenario, i.e., $\pi_B(y|x) = \pi_B(y)$. Then,

$$\sum_{\substack{y\in\mathcal{Y}\\ b\in\mathcal{B}}} \pi_B(y)V(a,b|x,y)P(b(y)|y,x,a)_{\hat{\sigma}^B_{a|x}} \leq \xi_B^{(x,a)}$$

Conjecture:
Alice ***always*** steers to the maximally certain state

J. Oppenheim and S. Wehner, Science, Vol. 330, No. 6007, 1072 (2010).

The uncertainty principle **determines** the strength of non-locality in QM (conjecture)

This conjecture was supported by an exhaustive analysis of the *-bipartite-* Bell inequalities known until Nov. 2010

(e.g. XOR games)

The conjecture also hold for Bell inequalities maximally violated by non-maximally entangled states*

*A. Acin, T. Durt, N. Gisin and J. I. Latorre. Quantum non locality in two three-level systems. Phys. Rev. A 65, 052325 (2002).

Counterexample

$$P(0,0|0,0) + P(1,1|0,0) + P(0,1|0,1) + P(1,0|0,1)$$
$$+P(0,1|1,0) + P(1,0|1,0) + P(0,1|1,1) \leq 3$$

Quantum violation ≈ 3.12

$$(x,a) = (0,0) \quad \rightarrow \quad \frac{1}{2}(P(b=0|y=0) + P(b=1|y=1)) \leq \xi_B^{(0,0)}$$
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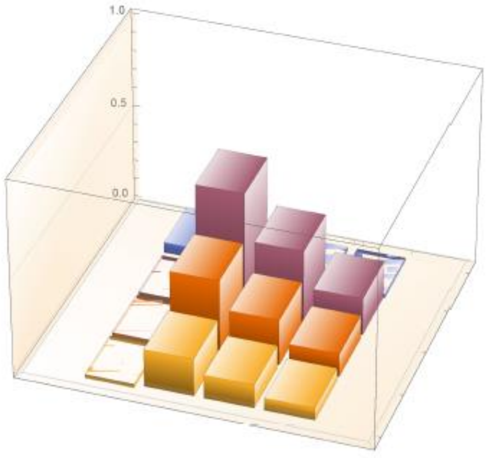
$$(x,a) = (1,0) \quad \rightarrow \quad \frac{1}{2}(P(b=1|y=0) + P(b=1|y=1)) \leq \xi_B^{(1,0)}$$
$$(x,a) = (1,1) \quad \rightarrow \quad P(b=0|y=0) \leq 1,$$

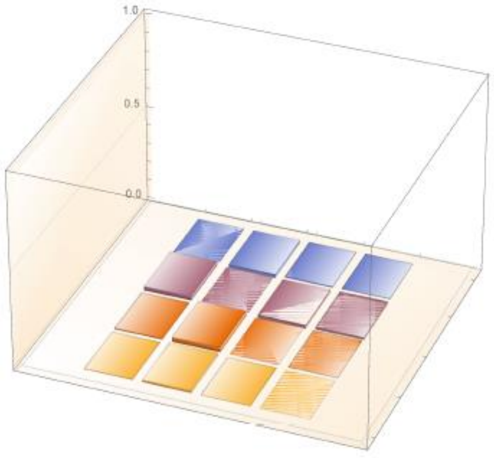
Only the fourth inequality is saturated.
Alice **cannot** steer to the maximally certain state

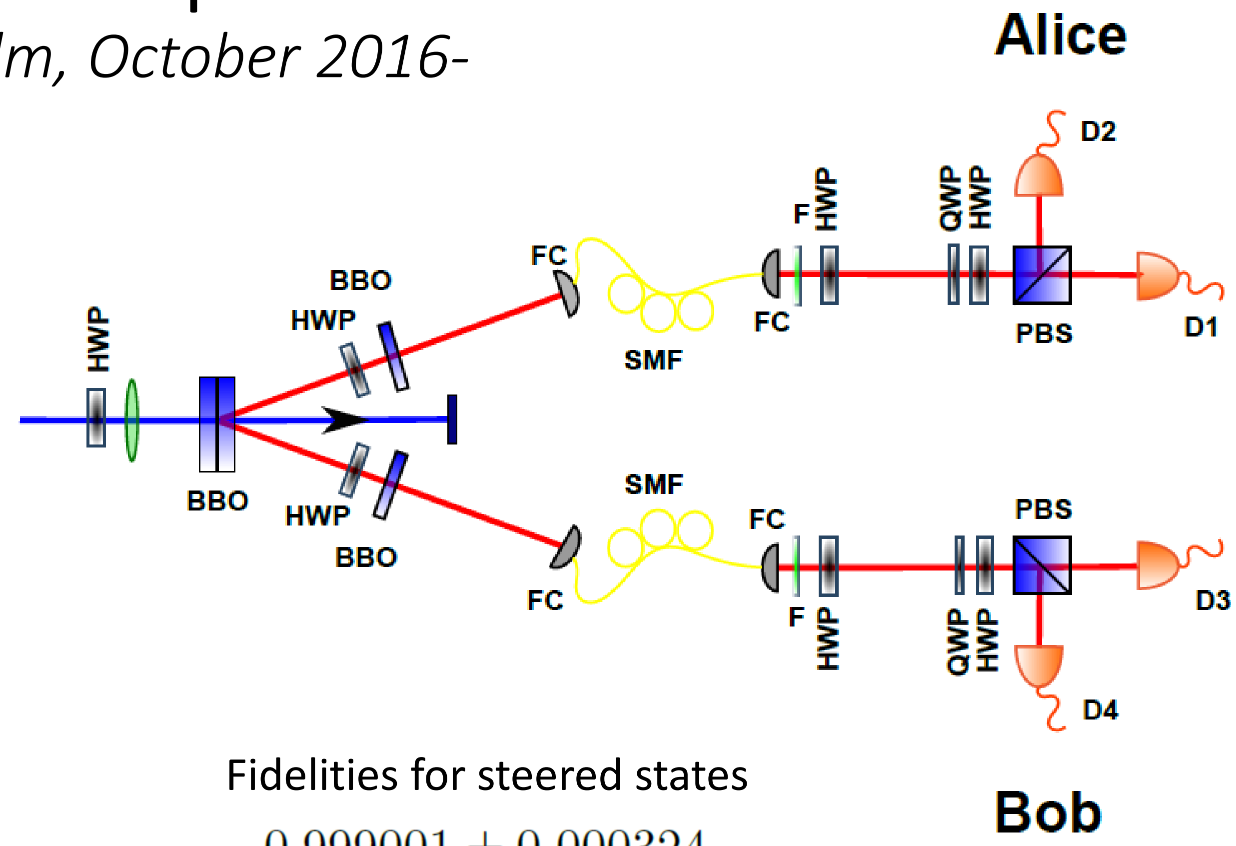
R. Ramanathan, D. Goyeneche, P. Mironowicz, P. Horodecki, The uncertainty principle does not entirely determine the non-locality of quantum theory, arXiv:1506.05100 [quant-ph]

Experimental implementation

-Stockholm, October 2016-







$$|\Psi\rangle = 0.248707|HH\rangle + 0.476081|HV\rangle + 0.806|VH\rangle - 0.248707|VV\rangle$$

$$F = \mathrm{Tr}(\rho_{\mathrm{theory}}\rho_{\mathrm{exp}}) = 0.993297 \pm 0.000867$$

Fidelities for steered states

0.999001 ± 0.000324

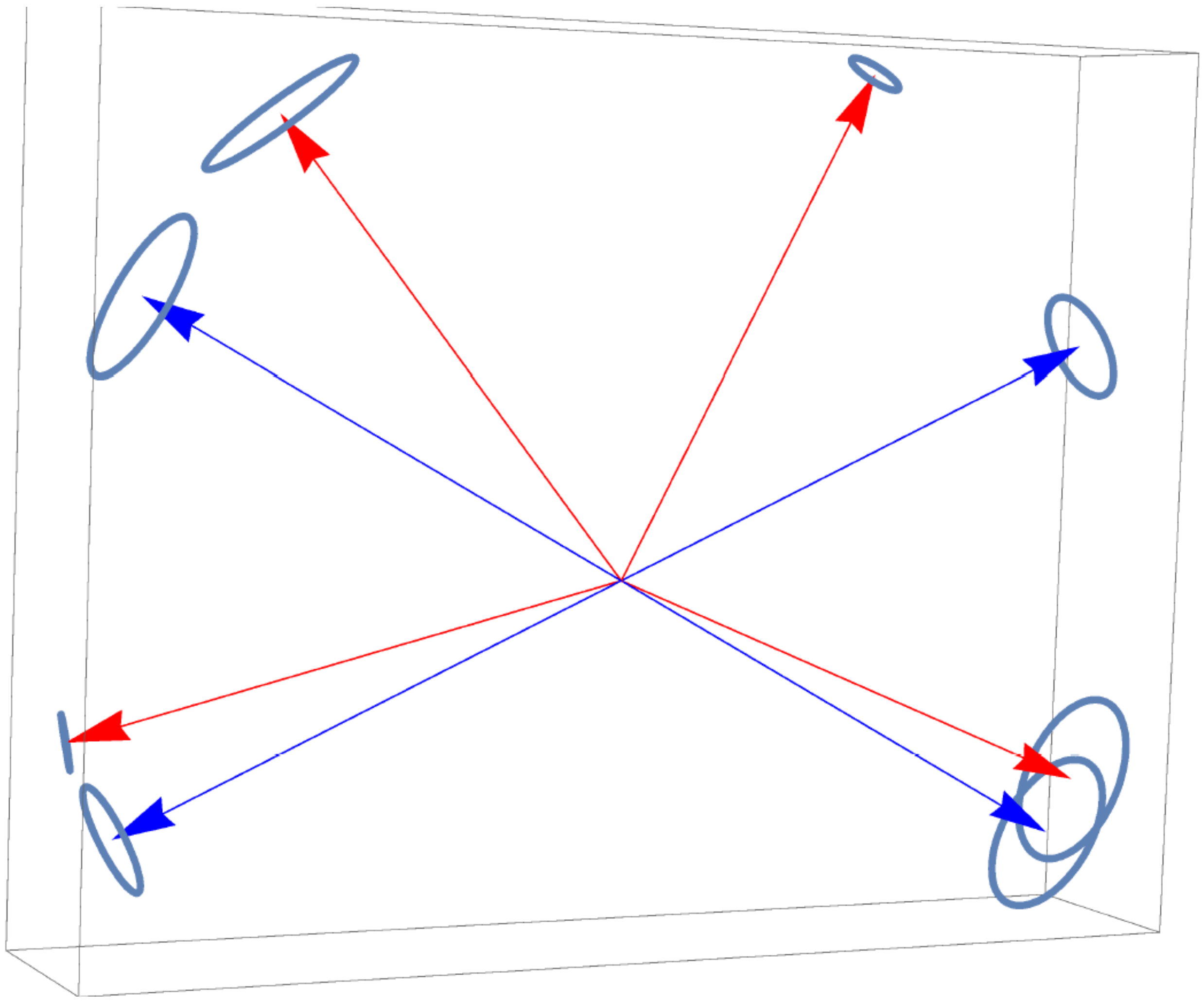
0.988826 ± 0.000806

0.989912 ± 0.000883

0.995704 ± 0.000383

Experimental implementation

-Stockholm, October 2016-



States that Alice steers

Optimal states for Bob

Resume

The uncertainty principle ***seems*** **to determine** the strength of non-locality in quantum mechanics (Oppenheim-Wehner, Science, 2010).

We have presented a Bell inequality where both **local uncertainty relations** and **quantum steering** play a fundamental role.

The **strength of the uncertainty relations** and the **degree of quantum steering** determine the strength of non-locality in quantum mechanics.

Schrödinger kitty

Thanks for your attention!

R. Ramanathan, D. Goyeneche, P. Mironowicz, P. Horodecki,
The uncertainty principle does not entirely determine the non-locality of quantum theory,
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