Environment induced collective entanglement in mesoscopic systems

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Outline

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Motivation and Aim of the Work

Experimental Facts:

- Repeated claims of "*macroscopic collective*" entanglement
  Quantum correlations between very large objects (*e.g.* clouds of 10^{12} Caesium atoms);
- Large systems can hardly be thought as completely isolated from their surrounding.

Motivation and Aim of the Work

Questions:

- What are good *quantum* collective observables for many-body systems?
- What is the role of an external environment?
  - Only a curse for quantumness of many-body systems?
  - Can, instead, an external environment generate *collective* quantum entanglement?
    (as it happens for entanglement between few particles?)

Entangled States of Bipartite Systems

Bipartite Quantum Systems

Compound system $S_1 + S_2$ (Hilbert space $\mathcal{H}_T = \mathcal{H}_1 \otimes \mathcal{H}_2$) is said to be entangled if

$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \,.$$

In general, an interaction Hamiltonian $H = \sum_i A_i \otimes B_i$ can generate entangled states from separable ones:

$$e^{-itH}|\varphi_1\rangle \otimes |\varphi_2\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \,.$$

In the density matrix formalism, one has that the state is entangled if

$$\rho_{ent} \neq \sum_i \lambda_i \rho_1^i \otimes \rho_2^i, \qquad \left(\lambda_i \geq 0, \sum_i \lambda_i = 1\right)$$

with ρ_j^i's are proper density matrices for the subsystems alone.

ENTANGLED STATES OF BIPARTITE SYSTEMS

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Bipartite Gaussian Continuous Variable Systems

Two Oscillators

$(\hat{X}_1, \hat{P}_1)$, $(\hat{X}_2, \hat{P}_2)$, such that

$$[\hat{X}_i, \hat{P}_j] = i\delta_{ij} \, .$$

If the state of the compound system ρ is Gaussian, *i.e* $\forall \hat{O} \in \{\hat{X}_1, \hat{P}_1, \hat{X}_2, \hat{P}_2\}$:

$$\mathrm{Tr}\left(\rho\, e^{i\alpha \hat{O}}\right) = e^{i\alpha \langle \hat{O} \rangle} \exp\left(-\frac{\alpha^2}{2}\Delta_{\hat{O}}\right)$$

with $\langle \hat{O} \rangle = \mathrm{Tr}\left(\rho\, \hat{O}\right)$, and

$$\Delta_{\hat{O}} = \mathrm{Tr}\left(\rho\, \hat{O}^2\right) - \langle \hat{O} \rangle^2 \, ,$$

then entanglement is fully characterized by means of a simple criterion.

R. Simon, *Phys. Rev. Lett.* **84** (2000)

MECHANICAL OSCILLATORS

Cloud of $N = 10^{12}$ Caesium atoms (B. Julsgaard *et al.*, *Nature* **413** (2001))

Collective spin operators J_x, J_y, J_z; spin polarized along z direction, with $\langle J_z \rangle \sim N \gg 1$
Define:

$$\hat{X} = \frac{J_x}{\sqrt{\langle J_z \rangle}}, \qquad \hat{P} = \frac{J_y}{\sqrt{\langle J_z \rangle}}$$

then, $\langle J_z \rangle \gg 1$,

$$[\hat{X}, \hat{P}] = i \frac{J_z}{\langle J_z \rangle} \rightarrow \text{classical value}$$

Re-scaled collective spin operators become position and momentum like operators.

AN OSCILLATOR ARISING FROM COLLECTIVE OPERATORS

Outline

1. Introduction
   - Motivation and Aim
   - Spin-$\frac{1}{2}$ chain
2. Collective Observables
   - Averages
   - Fluctuations
3. Entanglement of Two Spin-$\frac{1}{2}$ Chains
   - The Model
   - Collective Entanglement

N SPIN-$\frac{1}{2}$ PARTICLES

Consider one spin:

$$\text{Algebra of observables} \to M_2(\mathbb{C}) \to \begin{cases} \sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{cases}$$

N SPIN-$\frac{1}{2}$ PARTICLES

N two-level systems:

- Algebra of operators $\mathcal{A}_N = \bigotimes_{k=1}^{N} M_2(\mathbb{C})$:
- $\sigma_\alpha^{(k)} = \mathbf{1} \otimes \mathbf{1} \otimes \cdots \otimes \underbrace{\sigma_\alpha}_{k\text{-th site}} \otimes \cdots \otimes \mathbf{1}$
- Simple N-body state

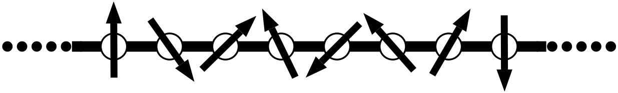
$$\rho_N = \bigotimes_{k=1}^{N} \rho = \rho \otimes \rho \otimes \rho \otimes \ldots$$

with ρ single spin state.

Expectation value of $O_N \in \mathcal{A}_N$: $\langle O_N \rangle = \mathrm{Tr}\left(\rho_N \, O_N\right)$

Outline

Classes of Observables



- Microscopic Observables:
  E.g. Magnetisation along 3rd axis at k-th site $\sigma_3^{(k)}$
- Macroscopic Observables:
  E.g. Mean-magnetisation of the system

$$S_3 = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \sigma_3^{(k)}$$

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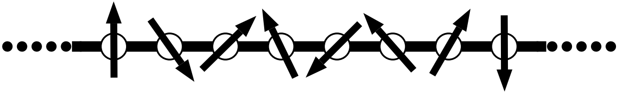

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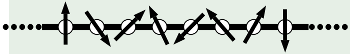
Average Operators

Averages

Spin-$\frac{1}{2}$ chain

$$\mathrm{Tr}\left(\rho_{\uparrow}^{N} X\right) = \langle \uparrow\uparrow \ldots \uparrow | X | \uparrow\uparrow \ldots \uparrow \rangle$$



$$S_{\alpha}^{N} = \frac{1}{N} \sum_{k=1}^{N} \sigma_{\alpha}^{(k)}$$

For $N \gg 1$:

$$\begin{matrix} S_1^N \\ S_2^N \\ S_3^N \end{matrix} \xrightarrow{\rho_{\uparrow}^{N}} \begin{matrix} S_1 = 0 \\ S_2 = 0 \\ S_3 = 1 \end{matrix}$$

In general:

$$\left\| \left[S_{\alpha}^{N}, S_{\beta}^{N} \right] \right\| \leq \frac{1}{N^2} \sum_{k,h=1}^{N} \left\| [\sigma_{\alpha}^{(k)}, \sigma_{\beta}^{(h)}] \right\| \sim \frac{1}{N}$$

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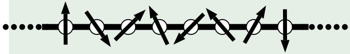

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Macroscopic Observables

Average Observables: A Law of Large Numbers

Considering $X_N = \frac{1}{N}\sum_{k=1}^{N} x^{(k)}$, all possible correlation functions

$$\mathrm{Tr}\left(\rho_N Y X_N Z\right),$$

($\forall\, Y, Z$ local observables) are such that:

$$\lim_{N\to\infty} \mathrm{Tr}\left(\rho_N Y X_N Z\right) = \left(\lim_{N\to\infty} \mathrm{Tr}\left(\rho_N Y Z\right)\right) \times \underbrace{\left(\lim_{N\to\infty} \mathrm{Tr}\left(\rho_N X_N\right)\right)}_{=\mathrm{Tr}(\rho\, x)}$$

so that, for $N \gg 1$

$$X_N \sim \mathrm{Tr}\left(\rho\, x\right) \mathbf{1}$$

Classical Description of Many-Body Systems

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Fluctuation Operators

Classification of Observables:

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$$S_3 = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \sigma_3^{(k)}$$

- Mesoscopic Observables: ??
  Can we learn something from classical physics?

Fluctuation Operators

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  Can we learn something from classical physics?

Fluctuation Operators

- Mesoscopic Observables:
  E.g. Fluctuation of magnetisation

$$F_{\sigma_3} = \lim_{N\to\infty} F^N_{\sigma_3} = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \left(\sigma_3^{(k)} - \mathrm{Tr}\left(\rho\,\sigma_3\right) \right)$$

Theory:

D. Goderis *et al.*, *Prob. Th. Rel. Fields* **82** (1989) 527; *Commun. Math. Phys.* **128** (1990) 533

A. Verbeure, *Many-Body Boson Systems* (Springer, London, 2011)

Experiments:

B. Julsgaard *et al.*, *Nature* **413** (2001);

H. Krauter *et al.*, *Nature Physics* **9** (2013);

S.L. Christensen *et al.*, *Phys. Rev. A* **89** (2014)

Fluctuation Operators

Fluctuations

Spin-$\frac{1}{2}$ chain

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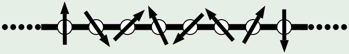

$$F_{\sigma_+}^{N} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \frac{1}{2}(\sigma_1 + i\sigma_2)^{(k)}, \quad F_{\sigma_-}^{N} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \frac{1}{2}(\sigma_1 - i\sigma_2)^{(k)}$$

$$\lim_{N\to\infty} \left[F_{\sigma_+}^{N}, F_{\sigma_-}^{N}\right] = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \sigma_3^{(k)} = \mathbf{1}$$

$$\lim_{N\to\infty} \mathrm{Tr}\left(\rho_{\uparrow}^{N} e^{\alpha F_{\sigma_-}^{N} - \bar{\alpha} F_{\sigma_+}^{N}}\right) = e^{-\frac{|\alpha|^2}{2}}$$

Fluctuation Operators

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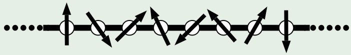

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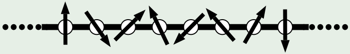

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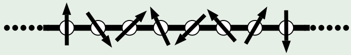

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Fluctuation Operators

Bosonic Behaviour

Spin-$\frac{1}{2}$ chain

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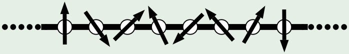
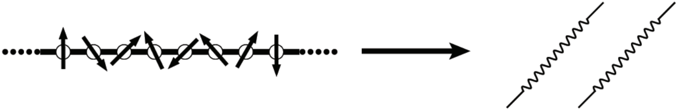


$$F_{\sigma_+}^{N}, F_{\sigma_-}^{N} \qquad a, a^\dagger \rightarrow [a, a^\dagger] = 1$$

$$e^{\alpha F_{\sigma_-}^{N} - \bar{\alpha} F_{\sigma_+}^{N}} e^{\gamma F_{\sigma_-}^{N} - \bar{\gamma} F_{\sigma_+}^{N}} \longrightarrow e^{\alpha a^\dagger - \bar{\alpha} a} e^{\gamma a^\dagger - \bar{\gamma} a}$$

$$\mathrm{Tr}\left(\rho_{\uparrow}^{N} \cdot\right) \qquad \langle 0 | \cdot | 0 \rangle$$ Fock Vacuum

The $N \gg 1$ Mapping: A Quantum Central Limit



$F^N_{\sigma_+}, F^N_{\sigma_-}$ $\longrightarrow$ $a, a^\dagger$
Ladder operators

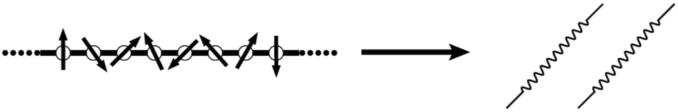
$e^{\alpha F^N_{\sigma_-} - \bar{\alpha} F^N_{\sigma_+}}$ $\longrightarrow$ $e^{\alpha a^\dagger - \bar{\alpha} a}$
Displacement Operator

$\mathrm{Tr}\left(\rho^N_\uparrow \cdot\right)$ $\longrightarrow$ $\langle 0| \cdot |0\rangle$
Fock Vacuum State

$$\lim_{N\to\infty} \mathrm{Tr}\left(\rho^N_\uparrow\, e^{\alpha F^N_{\sigma_-} - \bar{\alpha} F^N_{\sigma_+}} e^{\beta F^N_{\sigma_-} - \bar{\beta} F^N_{\sigma_+}} \ldots e^{\gamma F^N_{\sigma_-} - \bar{\gamma} F^N_{\sigma_+}}\right) = \langle 0| e^{\alpha a^\dagger - \bar{\alpha} a} e^{\beta a^\dagger - \bar{\beta} a} \ldots e^{\gamma a^\dagger - \bar{\gamma} a} |0\rangle$$

Collective Quantum Description of the Many-Body System

THE $N \gg 1$ MAPPING: A QUANTUM CENTRAL LIMIT



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COLLECTIVE QUANTUM DESCRIPTION OF THE MANY-BODY SYSTEM

Fluctuation Operators

Physical Meaning

The creator operator is such that

$$a^\dagger|0\rangle = |1\rangle \qquad \left(a^\dagger a|n\rangle = n|n\rangle\right);$$

this corresponds to the large N limit of

$$F^N_{\sigma_-}|\uparrow\uparrow\ldots\uparrow\rangle = \frac{1}{\sqrt{N}}\sum_{k=1}^N|\uparrow\uparrow\ldots\uparrow\downarrow^{(k)}\uparrow\ldots\uparrow\rangle\,.$$

Since, for $N \gg 1$,

$$a^\dagger a \propto \left(F^N_{\sigma_1}\right)^2 + \left(F^N_{\sigma_2}\right)^2$$

one can conclude that

$a^\dagger$ is creator of excitations of fluctuations perpendicular to direction 3.

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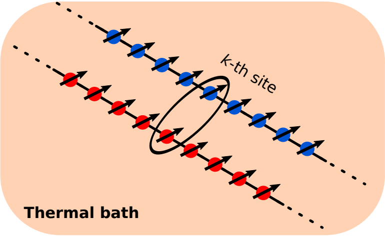
Two Spin-$\frac{1}{2}$ Chains



- Two non-interacting spin chains, at thermal equilibrium at inverse temperature $\beta = \frac{1}{k_B T}$

  Single-site state $\rho_\beta \propto e^{-\frac{\beta\epsilon}{2}\sigma_3} \otimes e^{-\frac{\beta\epsilon}{2}\sigma_3}$, $\quad \rho_\beta^N \propto \rho_\beta \otimes \rho_\beta \otimes \dots \rho_\beta$

$$\rho_\beta^N \propto e^{-\beta H_N}, \quad H_N = \sum_{k=1}^{N} \frac{\epsilon}{2}\Big(\sigma_3 \otimes \mathbf{1} + \mathbf{1} \otimes \sigma_3\Big)^{(k)}$$

- We focus on fluctuations of the operators:

$$\underbrace{\sigma_1 \otimes \mathbf{1}\,,\sigma_2 \otimes \mathbf{1}}_{\text{completely assigned to the first chain}} \qquad \underbrace{\mathbf{1} \otimes \sigma_1\,,\mathbf{1} \otimes \sigma_2}_{\text{completely assigned to the second}}$$

$$\underbrace{\sigma_1 \otimes \sigma_3\,,\sigma_2 \otimes \sigma_3, \qquad \sigma_3 \otimes \sigma_1\,,\sigma_3 \otimes \sigma_2}_{\text{supported by both chains}}$$

- By linear combination, one can construct fluctuations such that $N \gg 1$:

$$\begin{aligned} A_N^1 &\to a_1, & A_N^3 &\to a_3, \\ A_N^2 &\to a_2, & A_N^4 &\to a_4, \end{aligned}$$

such that

$$[a_i, a_j^\dagger] = \delta_{ij}\,.$$

A_N^1, A_N^3, and therefore a_1, a_3, result completely assigned to the first, respectively second chain.

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Thermal State of Fluctuations

- Four-mode "displacement" operator:

$$\mathbb{C}^4 \ni \mathbf{z} \to A_N(\mathbf{z}) = \sum_{i=1}^{4} z_i\, A_N^i, \qquad W_N(\mathbf{z}) = e^{A_N^\dagger(\mathbf{z}) - A_N(\mathbf{z})}$$

- Large N characteristic function:

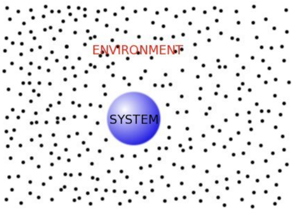
$$\lim_{N\to\infty} \mathrm{Tr}\left(\rho_\beta^N\, W_N(\mathbf{z})\right) = e^{-\frac{\|\mathbf{z}\|^2}{2}\coth\left(\frac{\beta\epsilon}{2}\right)}$$

- This is the characteristic function of the free four-mode thermal state:

$$R_\beta \propto e^{-\beta H}, \quad H = \epsilon \sum_{i=1}^{4} a_i^\dagger a_i$$

Open Systems Dynamics

System S immersed in thermal bath E



Assumptions:

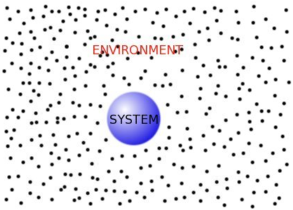
- no initial correlations S - E
- Weak S - E coupling
- Absence of memory effects

The reduced dynamics of the system observables O can be effectively described by Kossakowski-Lindblad type master equations

$$\mathcal{L}[O] = i\,[H, O] + \sum C_{\mu\nu}\left(V_\mu O V_\nu^\dagger - \frac{1}{2}\left\{V_\mu V_\nu^\dagger, O\right\}\right)$$

Open Systems Dynamics

System S immersed in thermal bath E



Assumptions:

- no initial correlations S - E
- Weak S - E coupling
- Absence of memory effects

The reduced dynamics of the system observables O can be effectively described by Kossakowski-Lindblad type master equations

$$\mathcal{L}[O] = i\,[H, O] + \sum C_{\mu\nu} \left(V_\mu O V_\nu^\dagger - \frac{1}{2} \left\{ V_\mu V_\nu^\dagger, O \right\} \right)$$

Open System Dynamics - GKSL Generator

For a d-dimensional system S:

$$\mathcal{L}[O] = i\,[H, O] + \sum_{\mu,\nu=1}^{d^2-1} C_{\mu\nu} \left(V_\mu O\, V_\nu^\dagger - \frac{1}{2} \left\{ V_\mu V_\nu^\dagger, O \right\} \right)$$

where:

- $H = H^\dagger$ coherent part of the dynamics;
- $\{V_\mu\}_{\mu=0}^{d^2-1}$ is a basis of the algebra of the system, $V_0 = \frac{1}{\sqrt{d}}\mathbf{1}_d$;
- C is the so-called Kossakowski matrix, $C \geq 0$.

Completely positive Markovian Dissipative Dynamics

The formal solution is:

$$O(t) = e^{t\mathcal{L}}[O] \equiv \sum_{k=0}^{\infty} \frac{t^k}{k!} \underbrace{\mathcal{L} \circ \mathcal{L} \circ \cdots \circ \mathcal{L}}_{k \text{ times}}[O], \qquad t \geq 0$$

$$e^{t\mathcal{L}} \circ e^{s\mathcal{L}} = e^{(t+s)\mathcal{L}}, \qquad t, s \geq 0$$

Microscopic Lindblad Generator

$$\mathcal{L}_N[X] = i[H_N, X] + \sum_{k,\ell=1}^{N} J_{k\ell} \sum_{\mu,\nu=1}^{4} D_{\mu\nu} \left(V_\mu^{(k)} X V_\nu^{(\ell)\dagger} - \frac{1}{2}\{ V_\mu^{(k)} V_\nu^{(\ell)\dagger}, X\} \right)$$

- with $\{V_\mu\}_{\mu=1}^4 = \{\sigma_+ \otimes \sigma_-,\ \sigma_- \otimes \sigma_+,\ \sigma_3 \otimes \mathbf{1}/2,\ \mathbf{1} \otimes \sigma_3/2\}$
- site-to-site statistical coupling: $\sum_{k,\ell=1}^N |J_{k\ell}| \sim N$
- $D = \begin{pmatrix} 1 & 0 & \gamma & \gamma \\ 0 & 1 & \gamma & \gamma \\ \gamma & \gamma & 1 & 0 \\ \gamma & \gamma & 0 & 1 \end{pmatrix}$ $\quad CP \begin{cases} |\gamma| \leq \frac{1}{2} \\ J \geq 0 \end{cases}$
- Thermal equilibrium state

$$\mathrm{Tr}\left(\rho_\beta^N e^{t\mathcal{L}_N}[O]\right) = \mathrm{Tr}\left(\rho_\beta^N O\right)$$

for any operator O.

Evolution of Fluctuations

- Fluctuation displacement operator:

$$W_N(\mathbf{z}) \to W(\mathbf{z}) = e^{A(\mathbf{z})^\dagger - A(\mathbf{z})} \ , \quad A(\mathbf{z}) = \sum_{i=1}^{4} z_i a_i \ ,$$

- We know that

$$\lim_{N\to\infty} \mathrm{Tr}\left(\rho_\beta^N W_N(\mathbf{x}) W_N(\mathbf{z}) W_N(\mathbf{y})\right) = \mathrm{Tr}\left(R_\beta W(\mathbf{x}) W(\mathbf{z}) W(\mathbf{y})\right)$$

- We would like to determine the map $\Phi_t[W(\mathbf{z})]$ such that:

$$\lim_{N\to\infty} \mathrm{Tr}\left(\rho_\beta^N W_N(\mathbf{x}) \boxed{e^{t\mathcal{L}_N}[W_N(\mathbf{z})]} W_N(\mathbf{y})\right) = \mathrm{Tr}\left(R_\beta W(\mathbf{x}) \boxed{\Phi_t[W(\mathbf{z})]} W(\mathbf{y})\right)$$

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DYNAMICS OF FLUCTUATION OPERATORS

THEOREM(BENATTI, F.C., FLOREANINI: ANN. PHYS. (BERLIN) 527, 639-655 (2015))

The evolution of fluctuations is implemented by a Gaussian dissipative dynamics:

$$\Phi_t[W(\mathbf{z})] = e^{\frac{\varphi(t)}{2}}\, W(\mathbf{z}_t)\,,$$

with $\varphi(t)$ at most quadratic in $\mathbf{z}$.

Heuristics:

- Condition $\sum_{k,\ell=1}^{N} |J_{k\ell}| \sim N$ seems to ensure *clustered* time-correlations between sites: we expect Gaussianity of the initial state not to be spoiled. This would imply, $N \gg 1$

$$e^{t\mathcal{L}_N}[W_N(\mathbf{z})] \to W_N(\mathbf{z}_t)e^{\frac{\varphi(t)}{2}}$$

Dynamics of Fluctuation Operators

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Heuristic Arguments

- Action of the generator $\mathcal{L}_N\left[A_N(\mathbf{z})\right] = A_N(M\,\mathbf{z})$, with M a proper 4×4 matrix; therefore,

$$e^{t\mathcal{L}_N}\left[A_N(\mathbf{z})\right] = A_N(\mathbf{z}_t), \qquad \mathbf{z}_t = e^{tM}\mathbf{z},$$

$$M = \begin{pmatrix} -1-i\epsilon & 0 & -\eta\gamma & \gamma\sqrt{1-\eta^2} \\ 0 & -1-i\epsilon & \gamma\sqrt{1-\eta^2} & \eta\gamma \\ -\eta\gamma & \gamma\sqrt{1-\eta^2} & -1-i\epsilon & 0 \\ \gamma\sqrt{1-\eta^2} & \eta\gamma & 0 & -1-i\epsilon \end{pmatrix}, \quad \eta = \tanh\left(\frac{\beta\epsilon}{2}\right).$$

- $\varphi(t)$ is constrained by the time-invariance of the state. Indeed, for $N \gg 1$,

$$\mathrm{Tr}\left(\rho_\beta^N\, e^{t\mathcal{L}_N}\left[W_N(\mathbf{z})\right]\right) = e^{\frac{\varphi(t)}{2}}\,\mathrm{Tr}\left(\rho_\beta^N\, W_N(\mathbf{z}_t)\right) = e^{\frac{\varphi(t)}{2}}\, e^{-\frac{\|\mathbf{z}_t\|^2}{2}\coth\left(\frac{\beta\epsilon}{2}\right)}$$

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$$\varphi(t) = -\left(\|\mathbf{z}\|^2 - \|\mathbf{z}_t\|^2\right)\coth\left(\frac{\beta\epsilon}{2}\right)$$

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Outline

Gaussian State of Fluctuations

- At $t = 0$ state of fluctuations prepared with a perturbation of the thermal equilibrium state, by single-mode squeezing the modes a_1, a_3 (same real squeezing parameter r)

$$R_0 = S_1(r)S_3(r)R_\beta\left(S_1(r)S_3(r)\right)^\dagger\,,\quad S_i(r) = e^{\frac{r}{2}\left(a_i^2-(a_i^\dagger)^2\right)}\;;$$

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- Entanglement measure $E(t)$ for two-mode Gaussian states.

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Entanglement Dynamics

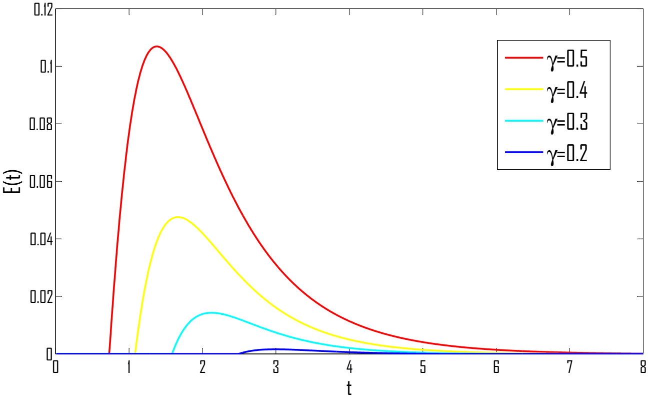

Figure : $E(t)$ for various values of the dissipative parameter γ, with fixed temperature $T = 0.1$ (T in unit of ϵ/k_B) and $r = 1$.

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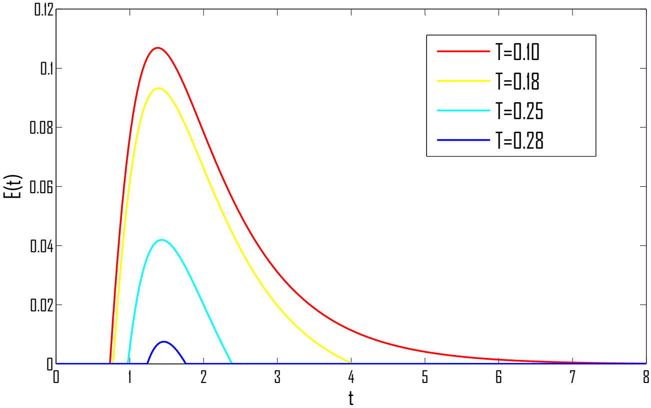
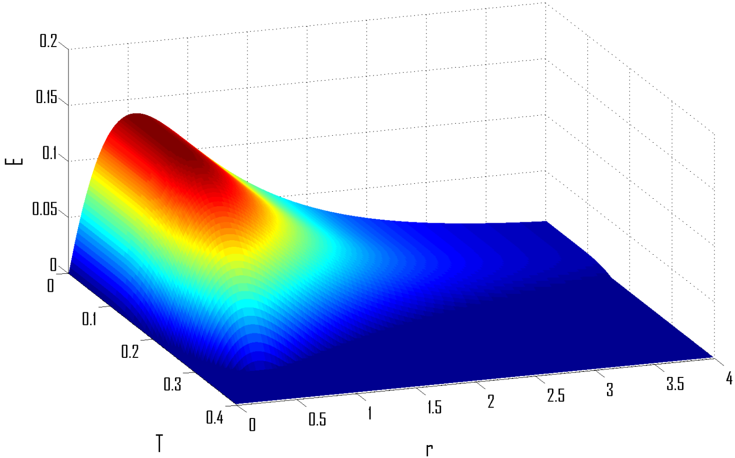


Figure : $E(t)$ for different values of the temperature, with fixed dissipative, $\gamma = 0.5$, and squeezing, $r = 1$, parameters.

Entanglement "Phase Transition"



Summary & Outlook

Also in a quantum framework, fluctuations play a relevant role in mesoscopic systems.

- Bosonic character of fluctuations irrelevant from the single-site algebra.

Their open quantum dynamics can be obtained starting from the microscopic one.

- Other kind of dynamics (*e.g.* Mean-Field Dissipative Dynamics);
- What if $\mathrm{Tr}\left(\rho_\beta^N\, e^{t\mathcal{L}_N}\left[W_N(\mathbf{z})\right]\right) \neq \mathrm{Tr}\left(\rho_\beta^N\, W_N(\mathbf{z})\right)$???

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Thank You

Thank You For Your Attention