Generating solitonic and vortex excitations in BEC via a potential sweep

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What will it be about

Introduction
  Solitons and vortices in BEC’s
  Other propositions

Our approach
  Solitons for noninteracting bosons
  Vortices
  Interacting bosons

Level crossings for interacting systems

Conclusions
Solitons

Phase imprinting method (Burger et. al. PRL 83, 5198 (1999).)

(a) $|\Psi_k|^2$

(b) $\Phi$

(c) Applied potential

BEC
Vortices - 1

Conversion between two components... (Matthews et. al. PRL 83, 24989 (1999).)

FIG. 1. (a) A basic schematic of the technique used to create a vortex. An off-resonant laser provides a rotating gradient in the ac Stark shift across the condensate as a microwave drive of detuning $\delta$ is applied. (b) A level diagram showing the microwave transition to very near the $|2\rangle$ state, and the modulation due to the laser rotation frequency that couples only to the angular momentum $l = 1$ state when $\omega = \delta$. In the figure, the energy splitting ($\sim 1$ Hz) between the $l = 1$ and $l = 0$ states is exaggerated.

FIG. 3. (a),(b): Two separate instances of the free evolution of a $|1\rangle$ state vortex in the magnetic trap. It is stable over a time long compared to the trap oscillation period (128 ms). (c) The free evolution of a $|2\rangle$ vortex is much more dynamic. It is seen shrinking quickly into the invisible $|1\rangle$ fluid and rebounding into fragments. Each column is from a single run, where time $t$ is referenced to the end of vortex creation ($t$ is the same for each row). The $|1\rangle$ and $|2\rangle$ state images appear different due to different signs of the probe detuning.
Laser stirring (rotating bucket) (Madison et. al. PRL 84, 806 (2000).)

FIG. 1. Transverse absorption images of a Bose-Einstein condensate stirred with a laser beam (after a 27 ms time of flight). For all five images, the condensate number is $N_0 = (1.4 \pm 0.5) \times 10^5$ and the temperature is below 80 nK. The rotation frequency $\Omega/(2\pi)$ is, respectively, (c) 145 Hz, (d) 152 Hz, (e) 169 Hz, (f) 163 Hz, (g) 168 Hz. In (a) and (b) we plot the variation of the optical thickness of the cloud along the horizontal transverse axis for the images (c) (0 vortex) and (d) (1 vortex).
Other propositions

▶ Collisions of condensates (W.P. Reinhardt, K. Burnett)
▶ Resonant Raman coupling (Marzlin, Zhang, Wright)
▶ Adiabatic passage (Dum, Cirac, Lewenstein, Zoller)
▶ bright/dark solitons, solitons in optical lattices...
Our approach

- Simple and elegant
- Similar idea for solitons and vortices
- Coherent method – uses an appropriate time-dependent modification of the potential
- Fast method – based on diabatic transitions
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Requirements

- Tightly focused laser beam for local potential modification
- Precise control over this beam intensity and direction
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Requirements

- Tightly focused laser beam for local potential modification
- Precise control over this beam intensity and direction
- Courage and luck
Noninteracting bosons

- Consider an elongated BEC in a harmonic (anisotropic) trap.
- One-dimensional approximation - oscillator units

\[
V(x, t) = \frac{x^2}{2} + U_0 \arctan(x_0(t)) \exp \left( \frac{-(x - x_0(t))^2}{2\sigma^2} \right).
\]

\[U_0 = 6.4, \ \sigma = 0.5\]
Level diagram

\[ U_0 = 6.4, \ \sigma = 0.5 \]
After a potential sweep.

Let \( p \) be a probability for a diabatic transition. For \( k \) out of \( N \) particles \( P(k) = \binom{N}{k} p^k (1 - p)^{N-k} \)

Single particle reduced density evolves at later times as

\[
|\psi(x, t)|^2 = (1 - p)\psi_0^2(x) + p\psi_1^2(x) + 2\sqrt{p(1 - p)} \cos(\omega t) \psi_0(x) \psi_1(x)
\]

In our case \( \dot{x}_0 = 0.1 \rightarrow p = 0.97 \)
Another example..

\[ U_0 = 13.4, \sigma = 0.2, \dot{x}_0 = 0.02 \rightarrow p = 0.99 \]

We can sweep the potential more than once...
We can create vortices too

- BEC in a 2D rotationally symmetric trap
- One has to "connect" the ground state with $L_z = \pm 1$ state

\[
H = \frac{p_x^2 + p_y^2}{2} + \frac{x^2 + y^2}{2} + U(x_0)e^{-\frac{(x-x_0 \cos \Omega t)^2 + (y-x_0 \sin \Omega t)^2}{2\sigma^2}}
\]

In the frame rotating with frequency $\Omega$

\[
H = \frac{\tilde{p}_x^2 + \tilde{p}_y^2}{2} + \frac{\tilde{x}^2 + \tilde{y}^2}{2} + U(x_0)e^{-\frac{(x-x_0)^2 + (y-x_0)^2}{2\sigma^2}} - \Omega L_z
\]

And again we sweep $x_0$ towards the center (along a spiral in LAB)
Example

\[ \Omega = 0.6, \quad \sigma = 0.2, \quad U_0 = 25. \]

\[ \dot{x}_0 = 0.036 \]

First and second sweeps →

\[ L_z = 1 \]

\[ L_z = 2 \]
Motion of the vortex

First sweep

\[ \Psi(\vec{r}, t) = \sqrt{\rho(\vec{r}, t)} \exp(i\chi(\vec{r}, t)) \]
\[ \vec{v} = \frac{\hbar}{m} \vec{\nabla} \chi(\vec{r}, t) \]
\[ \Gamma_C = \oint_C \vec{v} \cdot d\vec{l} = n\frac{2\pi\hbar}{m} \]
Vortices – Second sweep

\[ p_2 = 0.9997; \ p_0 = 0.0003 \]
Second sweep for small $\Omega$

For $\Omega < 1/3$ second excited state corresponds to $L_z = -1$!!
Stirring clockwise leads to counterclockwise vortex!!
Interacting bosons

- Whether avoided crossing survive in the presence of interactions?
- Whether diabatic sweeps are possible?

Brutal numerics: We solve now Gross-Pitaevskii equation:

\[
 i \frac{\partial \psi}{\partial t} = -\frac{1}{2} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi + g |\psi|^2 \psi; \quad -\frac{1}{2} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi + g |\psi|^2 \psi = \mu \psi
\]

for elongated BEC with effective 1D \( g = \frac{4 \pi N a_s}{a_h} \frac{\omega_\perp}{\pi \omega_z} = 50 \)
Vortices single sweep

\[ U_0 = 25, \sigma = 0.2, \]

\[ g = 100, \Omega = 0.23, \dot{x}_0 = 0.35 \]

\[ g = 500, \Omega = 0.12, \dot{x}_0 = 0.53 \]
Vortices second sweep

$g = 100, \ \Omega = 0.1$

$g = 100, \ \Omega = 0.25$

Observe large separation due to interactions
Level dynamics

\[-\frac{1}{2} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi + g|\psi|^2\psi = \mu\psi\]

Linear $H$

Nonlinear $H + g|\psi|^2\psi$

Loops - nonunique solutions for stationary Gross-Pitaevskii equation.
Level dynamics

\[-\frac{1}{2} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi + g|\psi|^2 \psi = \mu \psi\]

Loops - nonunique solutions for stationary Gross-Pitaevskii equation.
Explanation for loops

Let us sweep \( V(x) = \frac{x^2}{2} + U_0 \arctan(x_0) \exp \left( \frac{-(x-x_0)^2}{2\sigma^2} \right) \)

\[ g < 0 \text{ (attractive)} \]

\[ V_{\text{eff}} = V(x) + g |\psi|^2 \]

- Diabatic seems robust!!
- Loops indicate breakdown of Gross-Pitaevski effective description
- Loops = a manifold of tiny avoided crossings in many-body language

But this is another story...
Conclusions

- Novel method for solitons and vortices creation
- Simple and robust :-)
- a need for Optical access to BEC for a tightly focused, strong and well controlled laser beam :-(

People:

- Bogdan Damski (Los Alamos)
- Zbyszek P. Karkuszewski (independent)
- Krzysztof Sacha & J.Z. (Kraków)

Many body: J. Dziarmaga, Z. P. Karkuszewski, and K.