Dressed-atom model of lasing without inversion in the double-$\Lambda$ configuration

A. Karawajczyk, Z. Zakrzewski, and W. Gawlik

Instytut Fizyki, Uniwersytet Jagielloński, ul.Reymonta 4, PL-30-059 Kraków, Poland

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We present a dressed-atom theory of a four-level, double-$\Lambda$ atomic configuration interacting with a strong laser light which induces a superposition of two lower states. The presence of the cavity and of damping is fully taken into account and the conditions and characteristics of a possible lasing without inversion between the unperturbed states are determined analytically. The special case of the Zeeman-degenerate $J=1$ lower state is found to be particularly attractive for a possible practical realization.

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Although it is commonly believed that lasing in two-level systems requires population inversion, it is by no means generally true. Even in two-level systems the amplification [1,2] or lasing [3] may be obtained in the presence of a strong dressing field. Recently there has been much interest in lasing without inversion in three- and four-level systems [4–6]. These recent studies indicated the feasibility of laser action in two cases: (a) systems based on coherent population trapping [8] which can be regarded as a sort of redistribution of population in the appropriately transformed atomic frame [9] and (b) systems where the asymmetry between absorption and emission due to a quantum-mechanical interference in autoionization is utilized [4].

The "lasing without inversion" could be a very interesting alternative for obtaining coherent radiation in the cases where the usual methods are not applicable for technical or principal reasons like, e.g., in the far-UV regions or for strong resonant transitions in alkali metals. This paper presents a dressed-atom theory of the laser action without inversion in the double-$\Lambda$ configuration which fully takes into account the role of the laser cavity that was neglected in previous approaches.

We consider an ensemble of atoms with two ground-state sublevels 1 and 2 and two nondegenerate excited states 3 and 4 [Fig. 1(a)]. Sublevels 1 and 2 are optically coupled with state 3 by pumping fields of frequencies $\omega_1$ and $\omega_2$ and strengths characterized by the Rabi frequencies $\Omega_1$ and $\Omega_2$, respectively. State 4 can be coupled to 1 and 2 by a laser emission. The atoms in a collimated beam enter the cavity [Fig. 1(b)] whose two particular modes $\omega_{1c}$ and $\omega_{2c}$ are nearly resonant with the transitions $4 \rightarrow 1$ and $4 \rightarrow 2$ on which a possible laser action is expected. The evolution of the density matrix $\rho$ of the system is governed by the master (Liouville–von Neumann) equation (in the $\hbar=1$ units).

$$\dot{\rho} = -i [\mathcal{H}, \rho] + \mathcal{L}_A \rho + \mathcal{L}_F \rho \ ,$$

where

$$\mathcal{H} = \sum_{\mu} \left[ \sum_{i=1}^{4} E_i \sigma_i^{\mu} + \frac{\Omega_1}{2} e^{i(\omega_1 t - k_1 \tau)} \sigma_3^\upmu + \frac{\Omega_2}{2} e^{i(\omega_2 t - k_2 \tau)} \sigma_{23}^{\upmu} + \text{H.c.} \right]$$$ 

$$+ (g_1 e^{-ik_1 \tau} a_1^\dagger \sigma_{34}^{\upmu} + g_2 e^{-ik_2 \tau} a_1^\dagger \sigma_{24}^{\upmu} + \text{H.c.}) + \omega_{1c} a_1^\dagger a_1 + \omega_{2c} a_2^\dagger a_2 \ ,$$

with

$$\mathcal{L}_A \rho = 2 \sum_{\mu} \left[ \sum_{i=1,2} \gamma_{3i} (\sigma_i^{\mu} \rho \sigma_i^{\mu} - \frac{1}{2} \rho \sigma_i^{\mu} \sigma_i^{\mu} - \frac{1}{2} \sigma_i^{\mu} \rho) + \gamma_{4i} (\sigma_i^{\mu} \rho \sigma_i^{\mu} - \frac{1}{2} \rho \sigma_i^{\mu} \sigma_i^{\mu} - \frac{1}{2} \sigma_i^{\mu} \rho) \right],$$

$$\mathcal{L}_F \rho = 2 \Gamma \sum_{i=1,2} (a_i^\dagger \rho a_i - \frac{1}{2} a_i^\dagger a_i \rho - \omega_i \rho a_i a_i^\dagger) .$$

$a_i^\dagger$ and $a_i$ denote the creation and annihilation operators of the cavity photons of frequencies $\omega_i$ nearly resonant with $\omega_3, \sigma_i^{\mu} = |i \rangle \langle i |_\mu$ are the standard atomic operators for the $\mu$th atom placed in position $\tau_\mu$, and $g_\mu$ denote the coupling constant of the atom and the $\mu$th mode of the cavity. The second and third terms in Eq. (1) defined in Eqs. (3) and (4) are responsible for damping due to the spontaneous emission and the cavity loss, respectively.

The cavity bandwith [half width at half maximum (HWHM)] is denoted as $\Gamma$ while $\gamma_{3i}$ and $\gamma_{4i}$ stand for the free-space spontaneous emission rates from states 3 and 4 to sublevel $i$, respectively.

The theory developed, based on Eqs. (1)–(4), is capable of dealing with arbitrary parameters but we mostly elaborate here the specific situation where states 1 and 2 are degenerate Zeeman sublevels $m = \pm 1$ of the $J=1$ state and the transitions $1 \rightarrow 2, 1 \rightarrow 3$ are induced by a single, linearly polarized laser beam. In this case we have $E_1 = E_2, \Omega_1 = \Omega_2, \omega_1 = \omega_2 = \omega_c, \omega_3 = \omega_{2c} = \omega_c, g_1 = g_2 = g, \gamma_{41} = \gamma_{42} = \gamma_{4c}$, and $\gamma_{31} = \gamma_{32} = \gamma_{3c}$.

After transformation to the reference frame rotating with frequency $\omega$, one obtains a new effective Hamiltonian with new variables and energies shifted by $-(E_1 + \omega) / 2$, e.g., $E_4 = E_4 - (E_1 + \omega_3 + \omega) / 2$. It is natural to pass to the dressed-state basis where the only coupling...
After a usual semiclassical approximation for spontaneous emission we obtain equations for the macroscopic polarization \( S = \sum \alpha e^{i \phi} \sigma_{\alpha \dagger} \), the dressed-state populations \( \Pi_{a} = \sum_{\mu} \sigma_{\mu} \), and for the average photon field \( a = a_{1} = a_{2} \). In the equations, the flow of atoms in state \( \alpha \) into and out of the interaction region is taken into account by phenomenological rates \( \lambda_{\alpha} \) and \( \eta_{\alpha} \), respectively. These equations differ significantly from the standard laser equations [10] since the atom-cavity coupling constants are dynamically controlled in our case and, most importantly, because the existence of the trap state \( a \) and the coupling between \( b \) and \( c \) do not allow one to express the relevant populations solely in terms of inversion \( S_{3} = \Pi_{d} - \Pi_{c} \) but require more variables.

![Diagram](image)

FIG. 1. (a) Considered four-level system with the pump and generated laser fields. (b) Geometry of the experiment: atoms are pumped to the trap state by a wide pump beam before they enter the cavity.

left is that due to the expected laser action. The transformation from the bare \( |i\rangle \) to the dressed states \( |\alpha\rangle \) is \( |i\rangle = U_{i\alpha}|\alpha\rangle \) where \( i = 1, 2, 3, 4 \) and \( \alpha = a, b, c, d \) (with \( |d\rangle \equiv |4\rangle \)). This yields the effective dressed-atom Hamiltonian as well as the dressed-atom energies

\[
E_{a} = \frac{\Delta}{2}, \quad E_{b} = \frac{K}{2}, \quad E_{c} = -\frac{K}{2}, \quad E_{d} = E_{4}^{*},
\]

where \( K = (\Delta^{2} + 2 \Omega^{2})^{1/2} \) and \( \Delta = E_{1} - E_{3} + \omega \). From the explicit form of the unitary transformation matrix \( U \) and a dressed form of Eq. (2) it follows immediately that state \( a \) is decoupled from the expected laser action whereas the population of other levels may decay to \( a \). Thus \( a \) is a trapping state that accumulates the lower-state population while \( b \) and \( c \) are emptied by pumping and relaxation. This is the well-known phenomenon of coherent population trapping [8,9], which, as it was recently pointed out [5–7], should allow amplification on the transitions \( d \rightarrow b, c \). Such a gain occurs even though the population of level \( d \) is much smaller than the overall population of the lower state, i.e., without population inversion between the bare-atom states, yet with inversion with respect to the dressed states. The notion of “lasing with inversion” is, therefore, not precise as long as the particular reference frame is not specified.

We now concentrate on the laser emission from level \( d \). It is described by the equation for \( a_{i} \)

\[
a_{i} = -(\Gamma + i \Delta_{c}) a_{i} - \sum_{\alpha = a, b, c} i g U_{i\alpha} \sum_{\mu} e^{i q_{\mu}} \sigma_{\mu \dagger} a_{d},
\]

where \( \Delta_{c} = \omega_{c} - \omega \) and \( \sigma_{\mu \dagger} = |\alpha\rangle \langle \beta|_{\mu} \) are the dressed-atom operators. The last sum in Eq. (6) is the macroscopic polarization on the transition \( \alpha \rightarrow d \) and the \( e^{i q_{\mu}} \) factors reflect the dependence of the atom-cavity couplings on atomic phases that are random since atoms enter the interaction regions at random times.

With the energies given by Eq. (5) the lasing will take place to either one of the \( b \) and \( c \) dressed states. The particular laser transition can be selected by the cavity, provided its bandwidth \( \Gamma \) is smaller than the separation between the dressed levels \( (\Gamma < K) \). Below we choose to tune the cavity to the \( d \rightarrow c \) transition and take \( \Delta > 0 \), e.g., \( \Delta_{c} = E_{4}^{*} + K/2 \).

![Graph](image)

FIG. 2. (a) Calculated lasing regions plotted as a function of the pump-field parameters \( \Omega \) and \( \Delta \) for \( \lambda_{d} = 4 \times 10^{7}, \gamma_{3} = 10, \lambda_{b,c} = 0, \Gamma = 0.1, \eta = 0.1, \eta = 0.005 \), and for \( \gamma_{4} = 1(1), 10(2), \) and \( 11(3) \). Cavity detuning \( \Delta_{d} = -15 \) at \( \Delta = 0 \). (b) The dependence of the lasing region on efficiency of an incoherent excitation: \( \gamma_{4} = 10, \lambda_{d} = 10^{9}(1), 4 \times 10^{9}(2), \) and \( 3.2 \times 10^{10}(3) \); other parameters are as in (a).
To find the conditions under which the system can lase we look for nonzero stationary solutions of these equations in the form \( a(t) = ae^{-i\omega_L t} \) and \( S(t) = Se^{-i\omega_c t} \), where \( \Delta_L = \omega_L - \omega_c \). From the solutions we get easily the frequency of laser emission

\[
\omega_L = \frac{\omega_d \Gamma + \omega_c (\gamma_s + \eta)}{\Gamma + \gamma_s + \eta}, \tag{7}
\]

where \( \gamma_s \) is the dynamically controlled (i.e., intensity- and frequency-dependent) atom-cavity coupling constant, \( \gamma_{a0} \) represents damping from \( \alpha \) to \( \beta \), \( \gamma_c \) is a total damping rate of level \( \alpha \), \( \gamma_c = \gamma_c + \gamma_e \), and \( \Delta_{ac} = \Delta_e - (E_4 + K)/2 \). Factor \( G \) is positive when the expression in braces is positive too, so its detailed form is irrelevant to further discussion. If the pump beam is wide enough it is easy to obtain \( \lambda_{bc} \approx 0 \), which means that atoms that are not excited to \( d \) are fully trapped in state \( a \) before they enter the cavity. We thus obtain two following necessary conditions for \( |a|^2 > 0 \).

First,

\[
\gamma_c - \gamma_{cb} \eta + \eta > \gamma_{dc} + \gamma_{db} \eta. \tag{9}
\]

Condition (9) depends only on the spontaneous emission and escape rates and on the parameters of the dressing field. It shows the constraint between the loss rates for which the inversion between the dressed states \( \Pi_d \Pi_c \) is maintained. For example, for given values of \( \gamma_{ij} \), \( \Delta_e \), \( \Omega_e \), and \( \eta_e \), it determines the maximum value of \( \gamma_c \) for which the lasing is still possible [Fig. 2(a)]. This condition reflects the fact that the population of the lower lasing level has to be removed quickly enough to maintain the inversion between the dressed states, \( \Pi_d > \Pi_c \).

As a second condition for the laser action we obtain the threshold condition for the pump efficiency. It results from the requirement of positive value of the expression in braces in Eq. (8). For given values of both the pump and the atomic parameters, the threshold value of \( \lambda_d \) reduces with decreasing of \( \lambda_{bc}, \Gamma, \) and \( \Delta_{ac} \), which is consistent with intuition [Fig. 2(b)].

Similarly as in other papers [5,6], we found here, within the framework of the dressed-atom theory, that laser emission from one of the upper levels is possible when the strong field induces a linear superposition of the lower states, i.e., under conditions of the coherent population trapping. The earlier works on lasing without inversion [5–7] were limited to low-signal amplification of a weak probe field. In the present paper we fully analyzed the laser action within a cavity that can be tuned to resonance with any of the dressed-atom transitions and obtained realistic conditions of laser action depending on both the cavity and atomic characteristics. In particular, we observed the threshold reduction and frequency pulling, as well as selection between various possible dressed-atom eigenfrequencies by the cavity. The solutions obtained for the laser emission rate are stable in the Lyapunov sense for a wide range of relevant parameters.

The laser emission may occur in the considered configuration even though there is no population inversion between the upper and lower states. When passing to the dressed-state representation, we recognize, however, that such laser emission requires inversion between the dressed-atom states. For that reason the "lasing without inversion" obviously does not violate any fundamental principles and is consistent with a simple intuition.

Though the theory that was developed is quite general, for possible practical realizations we find particularly interesting the case where the lower state is the \( J = 1 \) state with the superposition of the \( m = \pm 1 \) degenerate sublevels acting as the trap. In this case a single linearly polarized laser beam can be used to prepare the atomic system for lasing action. Our dressed-atom theory can be easily extended to include the saturation effects and to the case of an atomic-gas sample, which is important for practical considerations.

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