Physics for fairer voting

Distributing votes fairly between European countries has been a major obstacle on the path to a European Constitution. **Karol Życzkowski, Wojciech Słomczyński and Tomasz Zastawniak** describe how physicists are helping politicians get round it

In October 2004 representatives of the European Union's 25 member states signed a treaty to establish a European Constitution. Since then the treaty has endured a bumpy ride, with citizens in France and the Netherlands voting against ratification last year. One of the sticking points has been the way votes are distributed between member states in the Council of Ministers, the main decision-making body of the European Union (EU). Physicists and mathematicians are now applying their statistical know-how to propose a solution to this problem.

The Council of Ministers consists of politicians from each member state who vote on behalf of their respective countries. To ensure that the influence of each country's vote reflects the size of its population without overwhelming the voice of smaller countries, the current voting rules – set out in 2000 in the Treaty of Nice – are based on a complicated system of "qualified majority vote". Each country is assigned a voting weight loosely based on its population, and approximately 72% of the total weight must be behind a proposal for it to be passed. In addition, at least 13 of the 25 EU states must support the proposal and the population of those states must exceed 62% of the total population of the EU.

During the drafting of the new constitution, ministers decided to simplify this system by dropping the voting weights. Instead, decisions would rely purely on the number of states voting for a proposal and on the proportion of the EU population comprised by those states. Under the constitution, a qualified majority would require at least 55% of member states and 65% of the total population to agree.

Although the proposal for the new constitution does away with the voting weights, which have no objective basis and tend to assign too much power to certain countries, it has flaws of its own. Large states, Germany in particular, would gain from the direct link to population, while small countries would derive disproportionate power from the increase in the number of states needed to support a proposal. The combined effect would sap influence away from medium-sized countries like Spain and Poland.

Is it possible to objectively design a voting system free of these deficiencies, in which each citizen of each member state would have the same power to influence the decisions made on their behalf? Can it be done in a way that is transparent, easy to implement, efficient to use, and will readily accommodate any future extensions of the EU (such as the inclusion of Romania and Bulgaria in 2007)? The answer, according to research carried out by the present authors, is "yes".

Bright idea
Physicists are using their knowledge of statistics to help make decision making fairer in the EU Council of Ministers.

Penrose's law

The flaws inherent in a system such as the EU Constitution were first outlined in 1952 by the British psychiatrist and mathematician Lionel Penrose, father of physicist Roger and mathematician Oliver. Penrose noted that if two rounds of voting were required for every decision, one on a per capita basis and the other on the basis of a single vote for each country, the system would be unfair as it would tend to favour large countries.

Penrose envisaged a fairer system based on a square-root law. As he pointed out, when general elections are held in a country with population N, a voter in a country with a larger population has less influence on the result than a voter in a smaller country. Since a particular individual's vote will only change the outcome of the election in the event of a dead heat, the voting power of a citizen is proportional to the probability of that result. Given that the votes are uncorrelated, Penrose calculated that this probability is proportional to $1/\sqrt{N}$.

In the case of the EU, the elected government then nominates a minister to vote on behalf of the country in the Council of Ministers, where the voting should ideally represent the views of every EU citizen equally, regardless of their nationality. It follows that each

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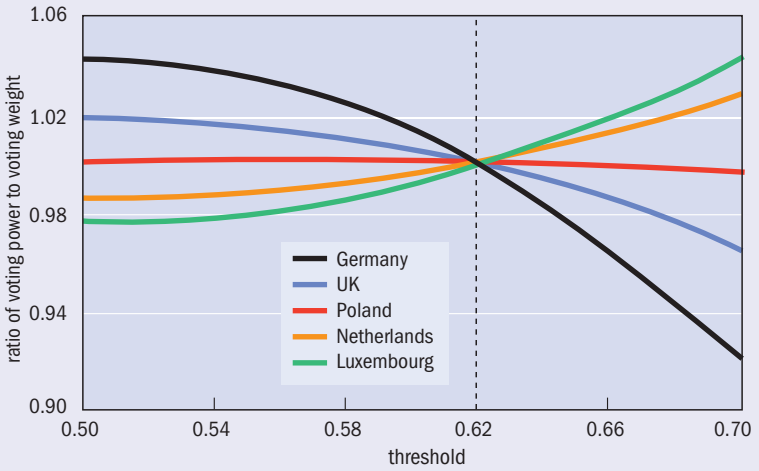
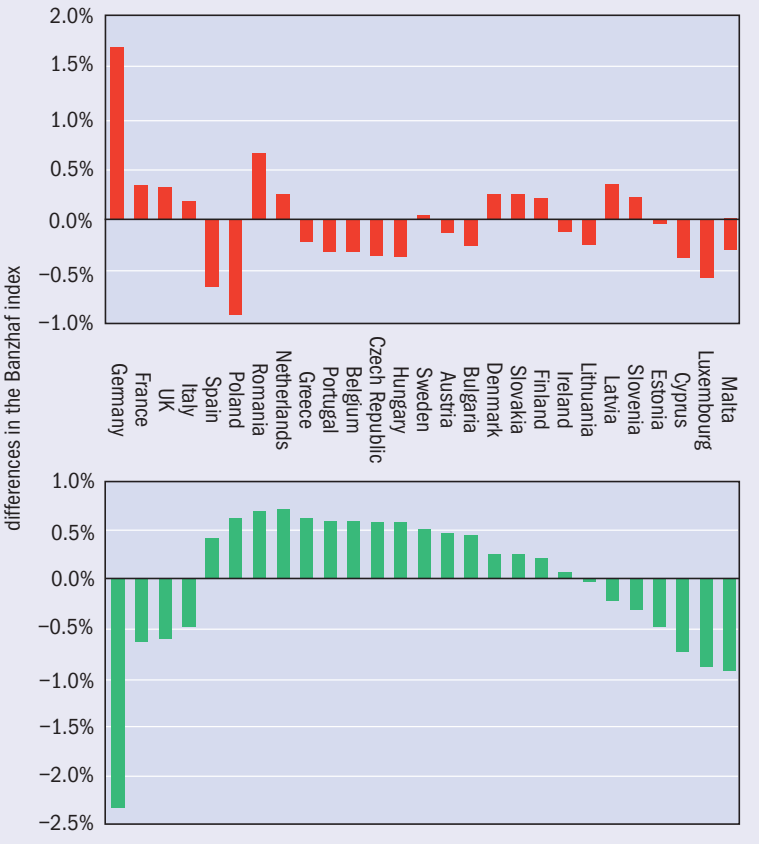
1 The critical point



By plotting the ratio of the voting power of each country to its voting weight against the threshold chosen for majority, a "critical point" emerges. At a threshold of 62% each country achieves a ratio of one, meaning its voting power is equal to its voting weight. Since the weights are chosen to be proportional to the square root of population, the voting power of each citizen is equal. With a lower threshold, larger countries have disproportionate power; while for a higher threshold, smaller countries have more influence.

2 The Jagiellonian compromise



If the Jagiellonian compromise was adopted, the voting power for each of the 27 EU states would change. The top graph shows the differences relative to the existing system in the Treaty of Nice and the bottom graph relative to that in the proposed EU constitution. The countries are ordered by population from largest to smallest. The Jagiellonian compromise, which allocates voting power according to the square root of population, restores some of the power to medium-sized countries (from Spain to Ireland) that would be taken away by the constitution. It is also apparent why it is called a compromise. Germany, for example, would gain considerable power under the new system compared with the Treaty of Nice, but not as much as it would if the proposals in the constitution were adopted.

country should have a voting power proportional to the square root of its population, $\sqrt{N}$, as this cancels out the factor of $1/\sqrt{N}$.

Penrose's square-root law – which has a direct physical analogy with the random walk of a diffusing particle – was first proposed as the basis of a voting system for the Council of Ministers in 1998 by Annick Laruelle of the Université Catholique de Louvain in Belgium and Mika Widgrén of the Turku School of Economics and Business Administration in Finland. Its application to EU decision making has also been investigated by Dan Felsenthal of the University of Haifa in Israel and Moshé Machover of the London School of Economics.

Although the Penrose law provides an objective procedure for allocating voting power to EU member states, simply giving each state a voting weight proportional to the root of its population does not guarantee that the country can exercise this power in making decisions. To see why, consider a hypothetical union of two states with voting weights 51% and 49%, respectively, in which proposals are adopted by a simple majority vote. The country with 51% of the voting *weight* will win every time – i.e. it has 100% of the voting *power*. It is therefore vital to distinguish between voting weight and voting power.

Indeed, a similar flaw persisted for 15 years with the council of the old European Economic Community: Luxembourg's single vote appeared to represent a greater weight than its small population merited, but, in fact, no possible coalition of the other countries required Luxembourg's vote to form a majority. The voting power of the country was effectively zero.

To generalize, a country only has voting power when its vote in a ballot is decisive: in other words, should the country switch its vote, the winning coalition would no longer satisfy the qualified majority condition. This concept can be formalized by considering all the possible coalitions that a country could take part in. The voting power can then be defined numerically as the proportion of these coalitions in which that country's vote is decisive. This definition was arrived at independently by Penrose in 1946 and the US lawyer John Banzhaf in 1965, and the proportion is often known as the Banzhaf index.

The 25 states of the EU can form more than 33.5 million possible coalitions, a figure which will increase four-fold when Romania and Bulgaria join next year. For a particular allocation of voting weights and choice of qualified majority rules it is possible to work out when a country holds a decisive vote for each of these coalitions. In other words, the voting power of a state, as measured by the Banzhaf index, can be calculated.

The Jagiellonian compromise

With the tools in place to measure the voting power of each state under a particular voting system, we can then try to choose a system that distributes these voting powers according to Penrose's square-root law.

In 2004 two of the current authors (KZ and WS at the Jagiellonian University in Kraków, Poland) proposed a voting system based on just one criterion: the voting weight of each member state should be allocated proportionally to the square root of its population, with decisions passed by the Council of Ministers

Is it possible to design a voting system in which each citizen of each member state would have the same power to influence decisions made on their behalf?

if the sum of weights of the countries voting in favour of a motion exceeds 62% of the total weight (arXiv.org/abs/cond-mat/0405396).

This system has a pleasing simplicity, but how was the magic number of 62% obtained? We first calculated how the voting power for each state depends on the threshold value chosen for a majority, and we observed a "critical point". As the threshold approaches 62%, the voting power of each country, irrespective of its size, converges on the ideal square-root value (see figure 1).

The Penrose law is thus fulfilled, and every citizen's voting power is equalized, with a simpler system than either of the official versions under discussion. Furthermore, any further enlargement of the EU would involve only one recalculation of the threshold for the qualified majority rule – which would become 61.4% in the 27-state EU. Indeed, the critical-point behaviour emerges for almost any number of member states or population distribution.

Our proposed voting system has stimulated considerable interest among experts in voting theory, and has been dubbed the "Jagiellonian compromise" by the media. Prior to the EU summit in Brussels in June 2004, an open letter in support of square-root voting weights in the Council of Ministers that was endorsed by more than 40 scientists in 10 European countries was sent to EU institutions and the governments of the member states.

The reaction of politicians has been varied, but inevitably depends more on how the Jagiellonian compromise affects an individual country's share of the vote (see figure 2) than on universal criteria such as simplicity and objectivity. When a similar system was put forward by Swedish diplomats in 2000, Sweden's prime minister Göran Persson said, "Our formula has the advantage of being easy to understand by public opinion and practical to use in an enlarged Europe…it is transparent, logical and loyal. Maybe that is why it does not please everybody." The former Irish prime minister John Burton has also made numerous positive references to voting systems based on Penrose's law, and the Jagiellonian compromise has been endorsed by a number of leading politicians in Poland and was scrutinized by UK government researchers in 2004.

If, as now seems increasingly likely, the European Constitution fails to come into force, the question of voting in the EU Council of Ministers will be revisited. The Jagiellonian compromise offers future negotiators a simple but objective system based on rational principles that grants equal voting power to all citizens of the EU.